

# The Basis of Atmospheric Mesoscale Dynamics and a Dynamical Method of Predicting Rainstorms



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By

Shouting Gao, Lingkun Ran  
and Xiaofan Li

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14.4.1. Time series of the ETS scores of the six-hour cumulative rainfall amounts forecast in 24 hour using the ensemble rainfall forecast method with dynamic factors (solid red lines) and the GFS model (dashed lines). The results shown are for forecasts of (a) over 10 mm/6 hours and (b) over 20 mm/6 hours for southern China ( $20^{\circ}\text{N}$ – $35^{\circ}\text{N}$ ,  $105^{\circ}\text{E}$ – $125^{\circ}\text{E}$ ) for 00:00 UTC, 2 June to 1 October 2010. (c) and (d) are the same as (a) and (b), respectively, but for 2012, and (e) and (f) are the same as (a) and (b), respectively, but for 2013.

## PREFACE

This book is an academic monograph that focuses on atmospheric mesoscale dynamics and the prediction of rainstorms, based on research results obtained by our team in the course of my academic career. The book includes some new research achievements made by our team in recent years.

I am grateful to my Ph.D. advisor, Professor Shiyan Tao for his constant advices in the past, and also thankful to Professor Duzheng Ye, Professor Qingcun Zeng, Professor Jifan Chou and Professor Chongyin Li for their encouragement during my career. Over a long period of time, I have also benefited greatly from the cooperation and discussions on scientific issues that I have had with my graduate classmate, Dr. Qin Xu. In recent years, Professor Qiushi Chen has helped me to advise graduate students and has also brought his ideas and methods related to the decomposition of physical processes to my research. I also benefited from discussing topics on non-uniform saturation with Professor Xingrong Wang.

The book is divided into 14 chapters. In Chapter 1, the basic dynamical parameters of the atmosphere and their physical meanings are discussed, with an emphasis on the Väisälä frequency and Richardson number. In Chapter 2, based on the governing equations of barotropic and baroclinic atmosphere, combined with a consideration of atmospheric mesoscale dynamics, the mesoscale potential vorticity equation and potential vorticity substance equation are derived. In Chapter 3, the physical quantities related to water vapor are introduced. In particular, the generalized potential temperature in a non-uniformly

saturated atmosphere is introduced and is shown to be a conserved quantity. This lays the theoretical foundation for the prediction of the area affected by a rainstorm. In Chapter 4, the vorticity equation and advection vorticity equation as well as the ‘frozen-in’ properties of the vorticity, streamline vorticity, and helicity are introduced. In Chapter 5, different forms of the divergence equation and potential divergence are discussed. In Chapter 6, the total deformation and deformation equation are brought in, and the interaction between the deformation, vorticity, and divergence is investigated. In Chapter 7, the generalized scalar frontogenesis function, which describes generalized scalar frontogenesis in a non-uniformly saturated atmosphere, and the trend of the frontogenesis function are described. In Chapter 8, the concept of potential vorticity and the potential vorticity equation are introduced. The concepts of second-order potential vorticity and generalized moist potential vorticity are developed. The moist potential vorticity anomaly with mass-forcing and the principle of the potential vorticity substance impermeability are demonstrated. In Chapter 9, based on a classification of the stability, several instability analysis methods, such as static instability, symmetric instability, and shear instability, are analysed. In Chapter 10, wave characteristics, wave polarization are given, and wave-action equation of three-dimensional inertial gravity waves and symmetric inertial gravity waves are discussed; a brief description of a method for identifying gravity waves from data is also presented. In Chapter 11, the mesoscale balance equation and its unbalance counterpart are investigated along with a related potential vorticity inversion technique. In Chapter 12, using a dynamic prediction method based on scalar field theory, the divergence, and deformation, as well as associated factors such as the potential vorticity, moist potential vorticity, and generalized moist potential vorticity, are used to identify and predict hot, humid weather in large cities during summer, as well as to trace and predict the movement of cyclones and to forecast torrential rainfall. In

Chapter 13, the convective vorticity vector, dynamic vorticity vector, ageostrophic Q vector, E vector, and wave-action vector are described along with related theories and dynamic prediction methods. In Chapter 14, rainstorm forecast methods with ensemble dynamic factors are discussed and demonstrated.

This book aims to deepen the understanding of mesoscale dynamics, to make the basic concepts clear, and to include new research results in the theory as far as possible. At the same time, the book also reflects the authors' own research interests and writing style, and strives to be innovative as well as being of academic value. In terms of content, the theories on which mesoscale dynamics relies are emphasized. For a long time, these concepts, including the theory of generalized potential temperature, convective vorticity vector, second-order potential vorticity, conservation of wave action, mesoscale balance equation, and generalized frontogenesis theory, have not received much attention, even though they provide the theoretical basis for the study of moist atmospheric processes. An improved method for analyzing gravity waves is also described in the book. One innovation is that some new types of vectors that can play an important role in dynamic prediction are proposed. Another feature of the book is the combination of theoretical knowledge and practical forecasting applications—this is seen in the method for forecasting rainstorms based on ensemble dynamic factors in particular. This method has been used in many operational provincial forecast centers in China. Some of the content of this book has been taught over a period of time at the University of Chinese Academy of Sciences, Zhejiang University, and Chengdu University of Information Science and Technology.

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—Shouting Gao



## ABBREVIATION AND MEANINGS OF THE TERMS USED IN THE STUDY

2D	Two dimensional
ADAS	ARPS data assimilation system
ARPS	Advanced Regional Prediction System
CISK	conditional instability of the second kind
CMA	China Meteorological Administration
CRISTA	Cryogenic Infrared Spectrometers and Telescopes for the Atmosphere
CSI	Conditional symmetric instability
CVV	Convective vorticity vector
DVV	Dynamic vorticity vector
ECMWF	European Center for Medium-range Weather
Forecast	
ETS	Equitable threat score
GFS	Global Forecasting System
GMPV	Generalized moist potential vorticity
GPS	Global Positioning System
GRAPES	Global/Regional Assimilation and Prediction System
IPV	Isentropic potential vorticity
JMA	Japan Meteorological Agency
K-H	Kelvin-Helmholtz
LIMS	Limb Infrared Monitoring of the Stratosphere
MLS	Microwave Limb Sounder
MOS	Model output statistics
MPVS	Moist potential vorticity substance
MVV	Moist vorticity vector

NCEP	National Centers for Environmental Prediction
NGMPV	Negative generalized moist potential vorticity
PV	Potential vorticity
QPF	Quantitative precipitation forecast
UCAR	University Center for Atmospheric Research
UTC	Coordinated Universal Time
WKB	Wentzel-Kramers-Brillouin
WKBJ	Wentzel-Kramers-Brillouin-Jeffreys
WRF	Weather Research and Forecasting

# CHAPTER ONE

## BASIC ATMOSPHERIC DYNAMIC PARAMETERS AND THEIR CONTEXT

In order to probe into the basic principles and dynamic structures related to the generation and development of mesoscale atmospheric systems, we must first understand some basic dynamic parameters that describe the mesoscale characteristics, such as the Rossby number, which is the ratio of the advective inertial force to the Coriolis force; the internal Froude number, which measures the atmospheric stratification effect; and the Richardson number, which represents the atmospheric stability. The specific contexts related to these parameters and how the parameters can be applied to the study of mesoscale atmospheric motion are also important, and so the first chapter of this book will introduce some important universal dynamic parameters.

### **1.1. Parameters Related to Rotation and Stratification and Their Dynamic Similarities**

The atmosphere is a rotating, stratified fluid that surrounds the Earth. (It is stratified according to temperature and humidity.) The effects of the Earth's rotation play an important role in the motion of the atmosphere. Therefore, it is important to ask at which scale the ambient rotation becomes an important factor in the atmospheric motion. To answer this question, we can begin with the ambient rotation rate, which is given by

$$\Omega = \frac{2\pi}{day} = 7.29 \times 10^{-5} / s . \quad (1.1.1)$$

If the time scale over which the atmospheric motion occurs is comparable with the time taken for one rotation of the Earth, or is longer, then this motion will be affected by the Earth's rotation. We, therefore, define the dimensionless quantity,

$$\omega_{\Omega} = \frac{2\pi/\Omega}{T} = \frac{2\pi}{T\Omega} . \quad (1.1.2)$$

If  $\omega_{\Omega} \leq 1$ , which corresponds to motion with a time scale of more than one day (24 hours), then it is necessary to consider the effect of the rotation. Usually, the ratio of the characteristic scale of the displacement,  $L$ , to its characteristic velocity,  $U$ , is used to represent the time scale of the atmospheric motion,  $T$ . The above dimensionless parameter is then defined as follows:

$$\tau = \frac{2\pi\Omega}{LU} = \frac{2\pi U}{\Omega L} . \quad (1.1.3)$$

If  $\tau \leq 1$ , consideration of the Earth's rotation is important.

Apart from the rotation, the atmospheric stratification is also very important. If the reference density of the atmosphere is  $\rho_0$ , the density change through a height  $H$  is  $\Delta\rho$ , and the basic velocity of the motion is  $U$ , then the corresponding potential energy change per unit volume is  $(\rho_0 + \Delta\rho)gH - \rho_0gH = \Delta\rho gH$ , and the basic kinetic energy is

$\frac{1}{2}\rho_0 U^2$ . The following dimensionless number can then be defined:

$$\sigma = \frac{\frac{1}{2}\rho_0 U^2}{\Delta\rho g H} \quad . \quad (1.1.4)$$

If  $\sigma \sim 1$ , then the stratification needs to be taken into account. Given such atmospheric stratification, for a disturbance to develop fully, the amount of potential energy required will be about equal to the kinetic energy. If  $\sigma \ll 1$ , then there is not nearly enough kinetic energy available in the stratified atmosphere for the disturbance to fully develop, meaning that the stratification plays a decisive role. If  $\sigma \gg 1$ , the changes in potential energy will have a small effect on the fundamental kinetic energy: in this case, the stratification is not important and the stratification effects can be ignored (Benoit Cushman-Roisin, 1994)<sup>1</sup>.

In atmospheric dynamics, there are some dimensionless parameters that are often used to represent the extent of the atmospheric rotation or stratification. These dimensionless parameters are obtained by making the atmospheric equations non-dimensional using the method of scale analysis. The parameters include the Rossby number, the internal Froude number, the Burger number, and the Richardson number.

The Rossby number,  $R_0$ , is the ratio of the advective inertial force to the Coriolis force: it can be expressed as

$$R_0 = \frac{U^2/L}{fU} = \frac{U}{fL} \quad . \quad (1.1.5)$$

The Rossby number can be used to represent the rotation of the atmosphere.

The internal Froude number,  $Fri$ , is the ratio of the advective inertial force to the buoyancy force, or of the kinetic energy to the gravitational potential energy:

$$F_{ri} \propto \left[ \frac{\rho_0 U^2 / L}{(\rho_2 - \rho_1) g} \right]^{1/2} = \frac{U}{\sqrt{g' L}}, \quad (1.1.6)$$

where  $g' = g(\rho_2 - \rho_1) / \rho_0$  and  $\rho_1, \rho_2$  are, respectively, the density of the upper and lower fluid layer;  $\rho_0$  is the reference density.

$$g' = g(\rho_2 - \rho_1) / \rho_0 = -g \frac{1}{\rho_0} \frac{d\rho}{dz} H = N^2 H,$$

Substituting this into (1.1.6) gives

$$F_{ri} = \frac{U}{\sqrt{g' L}} = \frac{U}{N \sqrt{H L}}. \quad (1.1.7)$$

For mesoscale systems  $L \sim H$ , giving  $F_{ri} = U / NH$ .

The internal Froude number clearly represents the atmospheric stratification effect. The definition of the Burger number is

$$B_u = \frac{f^2 L^2}{N^2 H^2}. \quad (1.1.8)$$

Furthermore, the Burger number is related to the Rossby number and the internal Froude number as follows:

$$B_u = \left( \frac{f^2 L^2}{N^2 H^2} \right) = \frac{\left( \frac{U}{N \sqrt{H L}} \right)^2}{\left( \frac{U}{f L} \right)^2} = \left( \frac{F_{ri}}{R_0} \right)^2. \quad (1.1.9)$$

For typical deep convection systems, usually  $L \sim H$ , which gives