The Principle of Stability within the Pantheon of Mother Ideas

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Ву

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# **FOREWORD**

As its sub-title suggests, this book is concerned primarily with philosophy. Its foundations, despite belonging to a tradition that goes back to, amongst others, pre-Socratic thinkers, Plato, Locke and Leibniz, are grounded in modern science, primarily mathematics, as well as physics and biology. This book is not intended for a specific audience but rather for well-informed readers in general. Following this tradition, it proposes an update of some of the main ideas which form the basis and shape the development of our current knowledge. Locke called them "simple ideas" and for mathematician Arnaud Denjoy, they were "mother ideas".

The notion of Stability naturally features in that ancient family whose *doyenne* is Movement. Stability, a stimulant for deeper investigation, has a key role to play in the advancement of knowledge. The aim of this book is partly to show the driving force of this notion, which can be discreet and even melt into the background, in all branches of intellectual activity. It appears in the term Stability Principle or, because of the latter's past, Plato's Principle.

This book is in two parts. The first specifically addresses mother ideas and the formulation of general facts, "primary observations", partly taken from a previous publication (*Topologie et Perception*, TP [9]). The second, "The stability principle and how affects shape humans" is, to a certain extent, more personal. It remains introductory, considering the breakthroughs to come in that vast undertaking – the study of the brain's workings.

Before I tackle the body of the text, let me revisit this issue of the role philosophy plays in our search for knowledge. At the start of the first chapter of TP, I spoke of Jean-François Revel's diatribe<sup>1</sup> against philosophers: Revel was indignant because the practice he was decrying added practically nothing to knowledge.<sup>2</sup>

<sup>&</sup>lt;sup>1</sup> In Revel, Jean-François. 1957. *Pourquoi des Philosophes ?* Paris: Juillard [32]

<sup>&</sup>lt;sup>2</sup> "Ils s'entreglosent" was Montaigne's way of describing pedantic discussions.

This diatribe holds true insofar that this practice is often limited to comments on the writings of ancient authors. The comments may well be learned but they provide no fresh insight into the present or future.

What do we actually mean by philosophy? Are we to understand it in the Greek sense? Is it the love of wisdom, meaning, in Plato's words, the love of knowledge, the search for universal knowledge that can turn humans into gods in the Greek sense of the word?<sup>3</sup>

In that case, we can assume that mathematical activity, for instance, conforms to this vision of philosophy. Mathematicians constantly seek to find out the properties of the objects they handle, to gain greater understanding of these objects, chief among which are the theories and universes into which they are structured, as well as the relationships and interactions between these universes. One might object that mathematics represents a narrow area of knowledge whereas, judging by Plato's and Aristotle's works and assertions<sup>4</sup>, the knowledge they invoke concerns all imaginable fields and all forms of expression of natural activity.

How did the Greeks and their predecessors go about this search for knowledge and what did their research consist in? A partial response to this question can perhaps be found in the potential and temporary description of a specific kind of philosophy, (there are others), one which I call "realist" or "positive", as Denjoy would say. Here it is, offered up for criticism:

Realist philosophy: this term describes a set of observations<sup>5</sup> and thoughts leading to the identification of analogies and to the formulation of often imprecise statements and conjectures on the common structure, behaviour and properties of varied and distinct fields. These may lead to further and deeper investigations in each of these fields, and to precise information being obtained, confirmed by experiments.<sup>6</sup>

<sup>&</sup>lt;sup>3</sup> Concerning what the Greeks and the Romans meant by philosophy, please refer to the prologue and part 1 of the book by Pierre Vespirini: *Lucrèce* [39]

<sup>&</sup>lt;sup>4</sup> For Plato's disciple, Aristotle, "wisdom is concerned with the cause of visible things" (Metaphysics, Book A, Chapter 7, 992a)

<sup>&</sup>lt;sup>5</sup> It would be amusing to count how many times Gauss uses the terms "observations" and "analogy", for instance.

<sup>&</sup>lt;sup>6</sup> This definition may be likened to the notion of hypothesis that I have just come across, in reading Hermann Weyl again. He first quotes physicist Ernest Mach for

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We shall see later that these statements are usually termed "simple ideas" or "mother ideas". The term "field" appears in this definition. It therefore underpins the idea that each of these fields, denoted  $F_i$ , generates a specific set,  $S_i$ , of observations and thoughts. There are therefore as many local fields as there are equally local philosophies. This is not to say that all  $S_i$  sets should be discrete, far from it. The way in which this family of sets is organized and structured brings to mind a lattice.

From this angle, mathematical philosophy can be seen as one of the local philosophies, and each of the objects of this discipline can in turn become the object of a specific philosophy. The object of a specific philosophy on the other hand,

whom "The essential function of a hypothesis consists in the guidance it affords to new observations and experiments, by which our conjecture is either confirmed, refuted, or modified, by which –in short– our experience is broadened" [40], page 157.

<sup>7</sup> The notion of Logic plays an important part in this local philosophy, mathematics. "Inference", a more erudite term for the notion of causality is also essential in this philosophy. The reality of the world does not provide just one causality but many causalities, the effects of which may be both positive and retrograde. Logic brings together different instances of local logic, each for a large part being an algorithmic formalisation of these causalities and their consequences. We tend to call "truth" a causality with properties of great stability. We are in the presence of local truths here.

The most elementary causality, which can also be described as primitive or naive, is the causality found in arithmetic. This is a particulate and infinite universe of objects called numbers. It is distributed, or stratified into sub-universes that are also particulate and infinite, whose elements display the same properties. There is total order among these sub-universes.

The first of these particulate and infinite sub-universes is that of numbers with identical properties, each named "1".

The second of these particulate and infinite sub-universes is that of numbers with identical properties, each named "2".

There might be a physical and very stable stacking process. When the process is implemented, any two objects stuck or welded together form an object in the second sub-universe. Here, the effective causality denotes the act of fusion between the two objects. To clarify, the notion of causality is a dynamic, potential or effective notion. More generally and more obviously, representations of physical phenomena by differential or by partial differential equations, using numerical systems, translate relations of causality. Their creators judge the latter to be stable, whether they rely on an actual physical act or solely on an act of thought.

An example of a very local "philosophy": "In the same way that classical Galois theory translates problems of etale algebras into problems of finite or profinite

can be characterized by the fact that its field is the sum of all local fields.

We can wonder about the history of these local philosophies, their origin and development and, in this context, the how and why of their potential influence on one another.

Lastly, we can wonder about the way these philosophies, analogies and conjectures were developed then expressed. What are the links and connections between the physical, biochemical, physiological and psychological mechanisms that underpinned this process? How does all this biological foundation operate and how was it established? We come back to the problem formulated by Aristotle:

"Holding as we do that, while knowledge of any kind is a thing to be honoured and prized, one kind of it may, either by reason of its greater exactness or of a higher dignity and greater wonderfulness in its objects, be more honourable and precious than another, on both accounts we should naturally be led to place in the front rank the study of the soul. The knowledge of the soul admittedly contributes greatly to the advance of truth in general, and, above all, to our understanding of Nature, for the soul is in some sense the principle of animal life.

Our aim is to grasp and understand, first its essential nature, and secondly its properties; of these some are thought to be affections proper to the soul itself, while others are considered to attach to the animal owing to the presence within it of soul. To attain any assured knowledge about the soul is one of the most difficult things in the world."

One of the preambles to the implementation of such an ambitious, Faustian, if not unrealistic project seems to be the development of a realist philosophy around which a degree of consensus might be achieved.

groups, the motivic Galois theory aims to assimilate an entire class of geometric problems to the classical representation theory of reduction groups and their Lie algebras. This philosophy has inspired many research projects, directly or indirectly, in the past thirty years or so." The term 'philosophy' describes here a methodological process, the use of an analogy bolstered by the perception of similar structures. Mathematicians often use the term 'philosophy' in its very narrow sense. Many old books feature this local characteristic, such as those written by Frege, Husserl and Russel. Indeed, the term 'philosophy' is used very frequently, even in the absence of a real link with their contents. Even Denjoy succumbs to this sloppy use of the word! (see *Mon œuvre mathématique, sa genèse et sa philosophie* in [18]).

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As a preamble, therefore, we ask this fundamental question: What is the body of mother ideas and primary observations that governs our reasoning, actions, decisions, judgments and choices.

The language of this book's original text –the writing was completed in June 2020– is French. The drafting style follows the way in which the author's thought process unfolds. The author is grateful to Hélène Wilkinson and thanks her warmly for her diligent translation of the text and excellent adaptation to the English language.

# PART ONE

# **MOTHER IDEAS**

Now grows
The One from Many into being, now
Even from the One disparting
Come the Many

Empedocles of Akraga

Then he who is able to do this has a clear perception of one form or idea extending entirely through many individuals each of which lies apart, and of many forms differing from one another but included in one greater form, and again of one form evolved by the union of many wholes, and of many forms entirely apart and separate

Plato, Sophist 253 d

Let us next proceed by investigating, not all the forms or ideas, lest we become confused among so many, but some only, selecting them from those that are considered the most important let us first consider their several natures, then what their power of mingling with one another is.

Plato, Sophist 254

# CHAPTER I

# THE NOTION OF MOTHER IDEA

Plato is to be credited with having defined and set out the methodological rules that characterize scientific discourse and development. These rules are set out in the final chapter of *Energie et Stabilité* [12], hereafter ES, from which the quotes above are taken.

All explanations rely on facts that are considered primary. They bear the traditional name of principles or axioms and here, they are also called Mother Ideas. What can these "intelligible realities"<sup>1</sup>, these primary data points be? And where do they come from in the first place?

Watchful observation combined with judicious conceptualisation bring these realities to light. Observation takes place over time and is renewed over successive generations. It addresses ever more varied and richer fields of activity, at increasingly broad scales, both towards the infinitely large and the infinitesimally small, instinctively or with the help of increasingly physical and intellectual tools. Primary data thus possesses the two qualities of relevance and generality. We tend to call it *universal*, that is, *stable across time and space*. It should however not be forgotten that this universality is

<sup>&</sup>lt;sup>1</sup> Plato would probably have made no great objection to the use of this expression, "intelligible forms": see this comment by Luc Brisson, author of the excellent *Platon* (Editions du Cerf, 2017, p. 136) [6]: "Le terme grec eîdos (qui vient du verbe eidénai signifiant « voir ») renvoie chez Platon à une chose vue, non par les yeux du corps, mais par ceux de l'âme, c'est-à-dire par l'intelligence, d'où la formule « forme intelligible ». À la différence d'un objet, la Forme est immuable: car elle ne naît ni ne périt, car elle est cause d'elle-même et surtout elle ne change pas. De plus, elle est universelle, en ce sens que partout elle est ce qu'elle est, ce qu'elle a été, ce qu'elle sera." (The Greek term *eîdos* (which comes from the verb *eidénai*, meaning "to see") refers in Plato's mind to things that are seen, not with the body's eyes, but with the soul's, in other words, with intelligence, hence "intelligible forms". Unlike an object, Form is immutable, it is neither born nor perishes, because it is its own origin and, above all, never changes. Moreover, it is universal, in the sense that it is always what it is, what it was and what it will be.)

relative to the various space-time observational fields we have been able to access and to their scope.

In addition, this data raises a number of questions. What is the reason for its existence? Is it well founded, what meaning and semantics does it carry? What can we deduce from the answers that may be put forward? How and where have they been shaped over time and by whom?

On the other hand, explanation, an intellectual object, usually turns out to be an assembly of various causes just as most purely physical, biological and social objects also appear in the form of more or less structured assemblies of similar sub-objects. What are these assemblies? How are they constructed, what are their potential or actual properties? Which of them, be they actually physical, functional or of another nature, can be described as universal?

Instinctive and intuitive answers, as well as those built up into theories, such as those relating to logic, have on occasion been proposed in reply to specific aspects of certain questions.

Philosophers of the 17th century, Malebranche and especially, Locke, who were analyzing the way in which thought processes occurred, introduced the term "simple idea". In theory, an idea that is not simple is the result of a combination of simple ideas. The choice of the adjective "simple" is linked to the apparently obvious nature of the corresponding idea, which is likely to be stated naturally, may not have required extensive thinking and is supported by solid consensus. In fact, the identification of simple and pertinent ideas can be the work of generations.

It is nonetheless remarkable that many basic ideas from which theories are developed, however learned they may be, do indeed appear to be those very so-called simple ideas. This is because, once they have been stated and established, some of them at least seem to be obvious, immediately understandable by anyone – sometimes making us wonder how we managed to

<sup>&</sup>lt;sup>2</sup> "These simple ideas, the materials of all our knowledge, are suggested and furnished to the mind only by those two ways above mentioned, viz. *sensation* and *reflection*. When the understanding is once stored with these *simple ideas*, it has the power to repeat, compare, and unite them, even to an almost infinite variety, and so can make at pleasure new complex ideas." [28] (Book I, Chapter II). Locke's successors, including Leibniz, adopted the expression "simple idea".

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miss them. This was precisely what Einstein felt. He had a thorough grounding in philosophy and had read Locke, who wrote at the start of *An Essay Concerning Human Understanding* (Book II, chapter XIII of the original 1690 edition):

"Though in the foregoing part I have often mentioned simple ideas, which are truly the materials of all our knowledge ..."

The feeling of finding great relevance in simple ideas is probably very common throughout the scientific community.

Denjoy, speaking of mathematics, appropriately called them "mother ideas"3:

"Ainsi que dans les sciences de la nature, le vrai problème de toute théorie est d'en découvrir les idées mères, celles qui, rattachant un effet capital (limitation, continuité, dérivabilité, analyticité, convergence, etc.) à une cause de la plus grande universalité possible, sont génératrices d'une nombreuse descendance de propriétés particulières" (As in the natural sciences, the real problem for any theory is to uncover the mother ideas, those which attach a crucial effect (limitation, continuity, integrability, analycity, convergence and so on) to a cause that is as universal as possible and which thus generate a significant lineage of specific properties.)

A very recent example expresses the same idea:

"The overall goal of this survey is to show how a few fairly simple ideas plus some standard techniques from deformation theory and Hodge theory explain a wide range of phenomena of the above kind"

writes Dutch mathematician Chris Peters in volume 4 of the *EMS Surveys* in *Mathematical Sciences* (July 2017).

Grothendieck was naturally of the same mind and he saw a childlike quality in the ability to identify simple ideas:

"C'est par le renouvellement que chacun a apporté dans plusieurs parties importantes de la mathématique, par des "idées" simples et fécondes, c'est-à-dire : pour avoir porté leur regard sur des choses simples et essentielles,

<sup>&</sup>lt;sup>3</sup> For Lazare Carnot, they took on the form of "idées primitives, qui laissent toujours quelque nuage dans l'esprit, mais dont les premières conséquences, une fois tirées, ouvrent un champ vaste et facile à parcourir. (primitive ideas that always left something of a haze in the mind. Once revealed, however, the main consequences of these ideas open up a vast field that is easy to explore"). [16] p. 3.

auxquelles personne avant eux n'avait daigné prêter attention. Cette capacité enfantine de **voir** les choses simples et essentielles, si humbles soient-elles et dédaignées de tous- c'est en **elle** que réside le pouvoir de renouvellement, le pouvoir créateur en chacun." (It is through renewal that each person makes contributions in a number of key elements of mathematics, through simple and fruitful "ideas", in other words, through turning their attention to simple and vital things, which no one else so far had bothered to look at. This child-like ability to **see** simple and vital things, however humble and despised, **this** is the repository of the power of renewal, the creative power that resides in each of us.) [22], p. 389.

It would be legitimate to query children's ability to perceive simplicity – that is, what is most important – amid quantities of data and complex phenomena. A degree of worldly knowledge and maturity would be lacking. And were it to be so, we would be entitled to ask why it took so long for just a few individuals throughout history to arrive at a statement of these simple ideas.

In fact, accepting them as such is far from being an immediate reaction. A hefty dose of knowledge, as well as experience and intuition, are often required to accept them totally and to use them wisely. Thus, a person who comes across such an idea for the first time may judge it to be simplistic, incomprehensible even, and may not immediately appreciate all of its embodiments, implications and manifestations, which may be quite subtle and sometimes far from obvious. They may even come to reject it. However, it can be hoped that their memory will preserve a trace of it and that a slow yet fruitful thought process will finally convince them that this "simple" idea, this mother idea, is pertinent.

In fact, to what extent is the term "idea" perfectly suited to defining the phenomenon it aims to describe. What about "notion", "concept", "fact" perhaps, or even "principle"? Might these terms not be equally suitable?<sup>4</sup>

<sup>&</sup>lt;sup>4</sup> In his apologetic *Thoughts* [30], Pascal, the archetypal religious man, does not explain what he means by "principles". Would he really include "mother ideas" in them (see note 9)? This is what he says about these principles: "J'en sais qui ne peuvent comprendre que, qui de zéro ôte quatre, reste zéro; les premiers principes ont trop d'évidence pour nous;" (I know even those who cannot understand that if four be taken from nothing nothing remains. First principles are too plain for us). Given that most children are now familiar with negative numbers, unlike Pascal, apparently, what would he say? Pascal says elsewhere: "Les uns tirent bien les conséquences de peu de principes: et c'est une droiture de sens....Il y a donc deux sortes d'esprit:

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We need to address here a problem in communication.

All mental constructs are hampered by a fundamental ambiguity: a conceptual construct can only be described by language, the terms of which belong to the very same construct. A new term can only be explained by using pre-existing terms, the meaning of which is clear and precise. If this is a primary term, there are –in theory– no pre-existing terms to explain it and the construct cannot be developed.

This means that we need to agree the conceptual value, the meaning of certain basic terms, from common experience. A change in terminology does not necessarily mean that the system is abandoned. Some notions, taken individually, or together, are interchangeable. There is a degree of latitude in the choice of premise, whenever a new model is established. In any case, the problem of arriving at the minimum number of words to write any discourse has been set. It would naturally be permitted to substitute a sentence with an equivalent expression and that expression would not belong to the basic corpus. We will circumvent that problem here. At the cost of allowing some obscurity in our thinking, we will on occasion content ourselves with self-evident truths and tautologies such as:

Definition: An object is an object.

This type of situation is obviously highly unsatisfactory for the mind. Chapter V will provide a more explicit definition of what an object is. It will however still mask the intrinsic difficulty in using primary terms.

l'une, de pénétrer vivement et profondément les conséquences des principes, et c'est là l'esprit de justesse; l'autre, de comprendre un grand nombre de principes sans les confondre, et c'est là l'esprit de géométrie. L'un est force et droiture d'esprit; l'autre amplitude d'esprit. Or, l'un peut bien être sans l'autre, l'esprit pouvant être fort et étroit, et pouvant être aussi ample et faible." (Some are able to draw conclusions well from a few premisses, and this shows a penetrative intellect.... There are two kinds of mind, the one able to penetrate vigorously and deeply into the conclusions of certain premisses, and these are minds true and just. The other able to comprehend a great number of premisses without confusion, and these are the minds for mathematics. The one kind has force and exactness, the other capacity. Now the one quality can exist without the other, a mind may be vigorous and narrow, or it may have great range and no strength.) The rest of the text on the distinction between a mind for mathematics and mind for subtlety is too long to include here. The distinction could describe two types of mathematicians, a classical analysis of mental faculties. In this second text, seen from the point of view of the mathematician, a "principle" is more in the nature of a theorem or proposition.

Returning to mother ideas specifically, one question will remain unanswered, that of the status of mother ideas falling under the category of observation. Leibniz [26], who would probably have placed these mother ideas in the "eternal truths" category, provided this answer:

"Nous trouvons en effet que dans le monde tout se fait selon les lois des vérités éternelles, qui n'appartiennent pas seulement à la géométrie mais encore à la métaphysique; c'est-à-dire tout se fait non seulement des nécessités matérielles mais encore selon des raisons formelles." (We find that everything in the world is governed by eternal truths that belong not only to geometry but also to metaphysics; that is to say not just of material necessity but also for formal reasons.)

Leibniz did not ask himself how these formal reasons were established nor how relevant they were.

In order to progress, in accordance with the spirit of the times, he introduced and referred to a personal and divine concept, in a way circumventing the problem, thus exposing our ignorance.

# CHAPTER II

# THE IDEA OF AN OBJECT (OR THING)

#### II.1. Introduction

Sensation is immediate and it is a fact both obvious and elementary that we are surrounded by objects. The term "object" is taken here to mean the same thing as "res", the term Lucretius uses, traditionally translated into English as "thing". Latin scholar Pierre Vesperini [39] describes res thus:

"Les choses, les *res* dont parle le poème ne sont donc pas seulement les choses « physiques » (pierre, feu, etc.) mais aussi tout ce qui « se passe », tout ce qui littéralement « a lieu » ..." (The things, the *res* mentioned in the poem are not just "physical" things (stone, fire and so on) but also everything that "happens", literally everything that "takes place"...)

### Vesperini drives the point further:

"De la sorte, ce sont toutes les *res* qui sont énoncées dans le poème: choses du ciel, de la terre, de la mer, des enfers, des villes, des forêts et des campagnes, récits mythologiques, grands évènements de l'histoire humaine, institutions politiques et religieuses, arts et techniques, maladies du corps et passions de l'âme, animaux, plantes, phénomènes étonnants, lieux mémorables de la terre habitée" (Thus, all these *res* feature in the poem: things in the sky, the earth, the ocean, the infernal regions, in towns, forests and countryside, in mythological tales, great events in human history, political and religious institutions, arts and techniques, bodily diseases and passions of the soul, animals, plants, amazing phenomena and memorable places on our planet.)

<sup>&</sup>lt;sup>1</sup> It is interesting to note that "thing" is re-introduced in the writings of some physicists. See for example G.F. Chew in his last article of 11 February 2019, Chew, G.F. 2019 "Chiral-Electromagnetic Gravitational Theory of Every 'Thing' Evolving Gelfand-Dirac Hamilton-Riemann Quantum Cosmology". Academia, April 20, 2019). Chew's "things" are far less general than Lucretius': they have mass. Chew stated he saw himself as a "thing".

The following are a few initial examples from the modern world, to illustrate the meaning attached here to the term "object".

#### Examples:

- 1. A public service strike, say, is a form of protest civil servants use to defend their rights and improve their lot. The strike actually exists, its effects can be felt, and therefore represents an object.
- 2. Isaac Newton's apple.
- 3. Objects such as those described in a dictionary or as understood by Piaget: "J'appelle objet un complexe polysensoriel, donc qu'on peut simultanément voir, entendre, toucher, etc, mais complexe polysensoriel qui, aux yeux du sujet, continue d'exister de façon durable, en dehors de tout contact perceptif." (I call "object" any multisensory complex, which can be seen, heard, touched, etc. simultaneously, and one that continues to exist lastingly in the eyes of the subject, even in the absence of any perceptive contact.)
- 4. "Classes and objects may (...) be conceived as real objects (...) existing independently of our definitions and constructions," says K. Gödel.

We accept this point of view.

The number of objects, of different types of objects, is, on the face of it, infinite. Each of their names in a specific language introduces the notion of cultural relativism. Any name describing an object in a given cultural context is steeped in the attached semantic halo, specific to the culture in which it appears. How can we be sure that two people, from different cultures, will arrive at the same interpretation of the names given to the same object in either culture? Communication can be improved by agreeing on the essential characteristics of an object, by ensuring that the definition of each named object carries, at least implicitly, the description of these common characteristics, for improved clarity and accuracy.

Moreover, given the infinity of objects mentioned above, classification needs to be introduced, to try and avoid drowning in this infinite world.

The classification process to be used here is based where possible on the deepest properties of objects, especially the component parts and models that govern their generation. Understanding this data will also help provide

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the material required to establish the definition of named objects. The following mother ideas are the first elements that will help us acquire and develop this knowledge. The order in which they are introduced cannot be random. They follow a temporal order on principle, depending on the time when they were revealed and accepted by the scientific community. The first mother ideas to be stated are linked to the deepest level of our very being's structure and operation.

# II.2 First descriptions of an object

The following are some of the apparently interlinked components which contribute to the classification of objects. They provide details on object *A* and describe it based on:

- 1. where it came from, how it was created, how it evolved, for instance,
- its architectural or structural configuration at a precise moment in time t. We define this configuration, K(A(t)), as the set of objects K<sub>i,A(t)</sub> contained in A(t) and which are the components of its internal makeup.

Definition 2.1: We call the structural graph of object A at time t – or, using a misnomer, its structure-graph G(A(t)). Its vertices represent the components of its internal makeup and its edges represent the functional links among these various components.

- 3. its *location*, that is, the D(A(t)) space-time domain it occupies at each moment, called the object's *spatial frame*.
- 4. two types of potentially measurable properties:
  - those  $s_j(t)$  whose variation affects its structure form a set S(A(t)) of size s. The various possible values of these properties are elements of  $\mathbf{R}^s$  or  $\mathbf{C}^s$  or of their generalizations.
  - and those  $k_i(t)$  with no effect on this structure. They form a set K(A(t)) of size k. The various possible values of these properties are elements of  $\mathbb{R}^k$  or  $\mathbb{C}^k$  or of their generalizations<sup>2</sup>.

These spaces are placed as fibers over D(A(t)).

<sup>&</sup>lt;sup>2</sup> Bruter, Claude P. 2007 "Du nouveau du côté des nombres" *Quadrature*, 66, 8-14. "http://arpam.free.fr/Du%20nouveau%20...%20Quadrature.pdf"

Definition 2.2: We call the data point in both sets S(A(t)) and K(A(t)) the state of the object at time t.

- 5. its environment  $D^{e}(A(t))$  at this time, that is, the sets of objects contained in the family of neighbourhoods of its domain D(A(t)).
- 6. A fifth way of describing an object is to take into account *its power* to change, which can be exerted on all the objects in this environment. This power therefore refers to another mother idea, that of change, which is the object of chapter IV, definitions 4.3 to 4.6 in particular. Chapter IV will examine several modes of change.

Any object A is the cause of possible changes and any change to A is due to one other object —at least. The potentialities for change and action directed at its environment are characteristic of what is called the object's "Me" in Chapter (see definition 4.7).

7. Last but not least among the elements characteristic of an object are those that are associated with its *stability properties*, the basis of its own ability to change, develop and be a source of force fields on its environment.

A dynamical system  $\Sigma(A(t))$  is associated with this object. A vector field V(A(t)), defining its local evolution on the manifold of state MA(t), is in turn associated with  $\Sigma(A(t))$ . This vector field takes into account the reciprocal influences exerted by the object and its environment.

This list of characteristics is also pertinent for objects that themselves constitute change, usually called events.

Classifying objects is anything but easy. Their set may be ordered but order can only be partial, since their definition relies on several criteria. We identify in particular an important class of objects, mental objects.

Definition 2. 3. We say that an object R(t) is mental when its spatial frame is contained in the nervous system of a living being.

We note in conclusion that the introduction of the general notion of an object offers a certain definition of the term "Nature" with this postulate:

PRIMARY OBSERVATION 0 (PO 0): Nature is composed of objects.

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Such a definition can be described as vague, to say the least, in the sense that a precise definition of what the term "object" covers is lacking, as are its potential meanings and embodiments. If we come back to the notion of object, as presented at the beginning of this chapter, Nature is itself an object. This is the classical aporia logicians face. Russell described it thus: is the set of all sets a set? A form of answer to this question is provided in Appendix 1.1.

# **CHAPTER III**

# THE IDEA OF PLACE<sup>1</sup>, PHYSICAL SPACE AND INTELLECTUAL SPACE

Where do we find objects? The question of their location is posed naturally, by first ascertaining what space they inhabit, where, failing a comprehensive description, we could at least detect their presence. The location question is inextricably linked to our abilities and tools, primarily sensory ones, which can detect this presence. All these tools are fashioned in the first instance from the data and manifestations of the physical world that shape us and that we can grasp. In humans, it is likely to be primarily the sense of sight, rather than the other senses, that governs the way we envisage space<sup>2</sup>, the space in which objects reside.

On a fundamental level, the basic space we need to take into consideration is the space which we feel is *continuous*, in which we move. Appendix 1.2 (*Les formes du continu*) reviews different aspects of the notion of continuity. We are *locally* capable of distinguishing those three special directions that are first, verticality, then, horizontality and finally depth, located in a different plane to that defined by the two previous directions. For us, space

<sup>&</sup>lt;sup>1</sup> See Appendix 1.2 for additional information on how mathematicians take the issue of space into consideration. *Mécanique Quantique*, *Et si Einstein et de Broglie avaient aussi raison*? [21], a book by Michel and Alexandre Gondran, provides further insight from the physics angle. *The Nature of Space and Time* [21], the "old" (1996) book by Hawking and Penrose does not mention it at all, although it does discuss its potential properties. The presentation by Marc Lachièze-Rey in the French edition is excellent.

<sup>&</sup>lt;sup>2</sup> The terms "principle"or "infinite" when used by the ancients, Anaximander for instance, often equate, as I see it, to the notion of physical space as we understand it today. This is what Theophrastus says in his *Opinions of the Natural Philosophers*: "Anaximander ... said that the principle and element of things is the Boundless, having been the first to introduce this very term 'principle'; he says that "it is neither water nor any other of the so-called elements, but some different, boundless nature, from which all the heavens arise and the world within them." See also what Aristotle occasionally says about the infinite in his *Physics*.

is locally three-dimensional. Its local character makes it highly Euclidian locally. Given the limited scope of our sensory abilities, space appears to us to always possess, in a first approximation and as far as the eye can see in any direction, those same *local properties*: *continuity*, *isotropy* and *dimension*. The term dimension is taken here in both its ordinary and mathematical sense, the first meaning size and scope and the second meaning primarily a direction and more generally a specific property.

Everyone knows the joke about the Englishman who lands at Calais and on seeing a woman with red hair, sends a text message (in today's parlance) to let his friends know of his pleasing discovery: "French women are redheads!". This tendency to generalize, which can be explained perfectly well, has provided a physical reality to the environment we find ourselves in, called here *physical space* or *physical continuity* – despite the fact that we are not able to describe the nature and scope<sup>3</sup> of this physicality precisely.

This is what Leibniz thought of space and his description naturally raises questions. In his preface to the *New Essays on Human Understanding* [28], he says:

"Il faut concevoir l'espace comme plein d'une matière originairement fluide, susceptible de toutes les divisions et assujettie même actuellement à des divisions et subdivisions à l'infini, mais avec cette différence pourtant, qu'elle est divisible et divisée inégalement en différentes endroits à cause des mouvements qui y sont plus ou moins conspirants." (we should think of space as full of matter which is inherently fluid, capable of every sort of division and indeed actually divided and subdivided to infinity; but with this difference, that how it is divisible and divided varies from place to place, because of variations in the extent to which the movements in it run the same way. 4)

The important properties discovered by physicists are too far out of the reach of our senses for us to arrive at a level where we are aware of space and can easily describe it orally. As a result, common mortals share the primary, elementary and simple vision of a continuous<sup>5</sup> and three-dimensional

<sup>&</sup>lt;sup>3</sup> It appears to be expanding. Theoretical physicists are currently advancing several theories. The progress expected in physical observation will likely lead to some of these theories being abandoned and to others being introduced...

<sup>&</sup>lt;sup>4</sup> Leibniz would have been attracted to the axiomatic nature of non-standard analysis. That being said, we don't think of a size entity as being the inverse of 2 to the power of aleph-zero (%o)!

<sup>&</sup>lt;sup>5</sup> The vision of a Democritus-style particulate space has disappeared. See for instance the conclusion of Appendix 1.7 on this topic.

physical space, with no further details.

We presume it to be substantial but can say nothing of the nature or origin of its substance. We can envision it as a form of infinitely fluid and clear soup with infinite "pep" potential and properties similar to those of superconductors. Physicists often refer to it as "ether" or "void", albeit naturally a fake void. In fact, the use of this inappropriate term should be banned. It is peopled by beings whose outline we barely perceive and whose origins are unknown to us: a local energy tensor that defines curvature and is linked to gravity, an amazing ability to transport electromagnetic energy in the form of light or waves of all frequencies, "audible" at the same time and in every point of this extraordinary space. More generally, this is a multiform wave-like agitation, which some of our instruments reveal as being more or less granular, such as photons that are massless but able to acquire mass. At best, we believe that, with suitable local "agitation", "lumps" will appear, in other words, pairs of certain elementary particles-antiparticles, initially electrons-positrons<sup>6</sup>, according to Dirac's first theory. A sexual analogy comes to mind: the kind of external penetration and energy supply in this environment breaks its internal symmetry –I will revisit this point– that of order 2: this is the order of the entity derived from near-perfect twins, the particle and its antiparticle, (bearing in mind that perfection<sup>7</sup> is an idea without a physical counterpart, should one study the matter in depth). The

,

<sup>&</sup>lt;sup>6</sup> 1) We probably get closer to the reality of physical space by also peopling it, at "normal" temperatures, with neutrinos, "virtual" quark-antiquarks pairs and by imagining it, at temperatures shooting to extreme values, as a perfect fluid, made of quarks. These are currently one-off elements, singularities in a way, conveying energy (mass, only because of the equivalence). This is the intimately clear fog from which our "particles" are said to emerge in phase transitions. As to the physical reality of quarks, this is Michael Atiyah's sensible point of view: "These point particle models are conceptually not very satisfactory, because a point is clearly an unphysical idealisation, a singularity of matter and charge density. An infinite charge density causes difficulties both in classical electrodynamics and in quantum field theories of the electron. Smoother structures carrying the discrete information of proton, neutron and electron number would be preferable." In other words, the quark phenomeon does indeed have physical reality, although we do not really know what form it takes.

<sup>2)</sup> The behaviour of certain electron-positron pairs, called positroniums, raises questions. More sophisticated observation methods would probably improve our understanding of the pairing process between a particle and an antiparticle. Neither particles nor antiparticles are all identical and why such and such a particle should pair up with such and such an antiparticle is yet to be discovered.

<sup>&</sup>lt;sup>7</sup> See footnote 4 Chapter IV.

introduction of chirality in theoretical physics is a way of bringing nuance to the perfection of symmetry. Just as we find the nature of physical infinitesimality (amongst other phenomena) very puzzling, we do not yet properly understand the particulate and wave-like nature of what we call photon, for instance (indeed, how can we appreciate billions of photos clustered around a point?) The existence of entanglement and its impact on apparently twin elements does not seem so odd in the light of the following analogy: it is usual for a pair of human twins placed in similar environments to also react in similar fashion. Although the progenitors are known in the case of living beings, it is still necessary, especially in the physical case, to establish the mathematical superstructure and the exact nature of the physical field underpinning entanglement and the sharing of information, and also to guess their origin and purpose...

One of the remarkable properties of physical space is due to our ability to move within it in many different ways (see section IV.3.2), and our ability to move at will certain objects, for instance a glass or a plate, altering or not their properties.

The notion of *res*, or "object" taken in its general sense, encompasses ideas, in the usual sense of the word, and all our mental constructs. Constructs come from our observation of the environment through our senses, they are shaped and developed by our mental mechanisms, either spontaneously or through learning, and they are all anchored physically in our brain. As such, they belong in the physical space. It is naturally only very partially possible for us to access this physical anchorage.

Specifically, we are unable, at least for now, to attribute metric properties to mental constructs. We say that they form a specific "space", *the intellectual "space*".

This is the space of our "mental representations", representation being a critical activity which I will be returning to. The intellectual space includes representations of objects to which metric data has or has not been attributed, forming a sub-set RE1, called of first level, level 1. It also contains the sub-set of level 2 representations, developed as set RE2 of the representations of RE1 elements, a basic sub-set of the so-called abstract representations, which in turn develops into more elaborate strata as regards abstraction and generality.

<sup>&</sup>lt;sup>8</sup> See the note, Ignoramus, at the end of this chapter.

## **Note: Ignoramus**

We can but marvel at Nature's extraordinary prowess. It shows up the weakness of our understanding, through our conceit, naivety or foolhardy intelligence. We try to draw up models from what we observe, the better to find explanations. But we sometimes forget that we are reliant on our observational and representational tools, be they physical or intellectual, and on our thinking habits. In other words, we occasionally mistake the model for reality and fall into error as a result.

The idea of associating a physical spectrum to a spectrum generated by a suitable mathematical operator is obviously attractive. It is even likely that the mathematical denomination has adopted the one physicists use. We proceed by analogy, that is, we imply the presence of a common mechanism, revealed by the presence of the same morphology. Two excited states in the atom are represented by two specific values on the spectrum. Yet, is the sudden energy jump observed by physicists really that discontinuous in physical reality, as Niels Bohr conjectured? Can we not track it step by step, by following all the components' movements? Physicists have successfully tackled the task<sup>9</sup> (see this extract from the footnote: The experimental results demonstrate that the evolution of each completed jump is continuous, coherent and deterministic). We are not yet totally capable of understanding the phenomenon properly, or of describing it in its entirety, especially the way electrons are generated and distributed in fairly stable "layers" and how transition from one layer to the next occurs. All this has justified an initial description, which is quantum by nature, in other words, discontinuous, and the design of models appropriate for this level of sophistication in observation. However, the situation seems to be evolving.

No one has in depth knowledge of electrons and photons<sup>10</sup>. We do not describe elementary particles—never mind quasi-particles—for what they are in their entirety but rather for their accessible and "sensitive" properties (charge for instance), for their effects and for part of their structure, through

<sup>&</sup>lt;sup>9</sup> Read (click): "the life-or-death quantum jump is not quite instantaneous after all" and "physicists working on quantum computers".

<sup>&</sup>lt;sup>10</sup> Hence the difficulty in understanding physical phenomena at this infinitesimal level and physicists' varied opinions on their interpretation. On this topic, see for example: Musser George, 2020 "Paradox puts objectivity on shaky footing", *Science* 369, 889-890, August 28 2020, and related articles. Things are likely to become clearer when we achieve more sophisticated observations.

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symmetries. Are they, by any chance, somewhat similar to a kind of whirlwind, in the spirit of Descartes or even Kelvin? These whirlwinds already feature in ancient philosophies and they can now be seen in skyrmions. Have we not endowed them with a kinetic or angular momentum called spin? Could we not consider them as almost one-off eyes of very local cyclones, the very locally solidified eyes, as is the case in phase transition? This is, in a way, the point of view Louis de Broglie reasonably adopted when he broke down the photon's wave u into " $u_0 + v$ , où  $u_0$  représente la très forte concertation d'énergie très localisée et où v représente tout le reste de l'onde" ( $u_0 + v$ , where  $u_0$  represents the very high concentration of highly localised energy and where v represents the rest of the wave).

This basic idea, and its implications, partly set out in two books, *Nouvelles perspectives en microphysique* [7], *La réinterprétation de la mécanique ondulatoire* [8] is an attractive one. These two books make a clear distinction between corpuscular and wave-like aspects. As de Broglie would have it, we can conjecture that any object possesses these two properties but when we write them, we encase them in inverted commas: "corpuscular", "wave-like". It would appear that corpuscular properties, at the macroscopic level, are shown by the presence of mass and wave-like properties by the presence of regular internal movements: *they both ensure and signal the object's stability*. In order to understand this, for example in humans, all biorhythms and vibratory phenomena, in all the body's component parts, need to be taken into consideration. This is a huge undertaking, which has hardly ever been tackled. The presence and weight, as well as the potential breakup of a wave into solitary waves, are still unknown.

I see chemistry, then biochemistry, then biology and organics as the deployment of the physical world's properties and modus operandi, as well as its potentialities, which are difficult to see and/or not fully expressed. Which is why, on the topic of quantum leaps, I enjoyed reading—and now sharing—the following extract from *Science* on the morning of 8 June 2019:

"One of the most intriguing conclusions from Soldatov *et al.* is the identification of specific steps involved in neural crest differentiation, in which progenitor cells undergo binary choices between possible fates as a result of their cellular history. This history is defined by the internal and external events that the cell has experienced, such as the autonomous activation of genes as well as signals from neighboring cells. Progenitor cells initially coactivate gene expression programs that lead to competing cellular fates (see the figure). These mutually exclusive cell fate programs then compete with each other. This competition is determined by differences in gene expression