Theory and Methods of Vector Optimization (Volume Two)

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VOLUME 2. VECTOR OPTIMIZATION MODELLING OF ECONOMIC AND TECHNICAL SYSTEMS

The second volume of this work presents the practical use of the theory and methods of vector optimization in the field of mathematical modelling and the simulation of economic and technical systems. It is divided into two parts: economic and technical systems.

PART 1. VECTOR OPTIMIZATION IN THE MODELLING OF ECONOMIC SYSTEMS

The first part of the second volume presents research into the mathematical modelling of economic systems on the basis of the theory and methods of vector optimization.

The first part includes three chapters.

The first chapter deals with issues related to the theory of the company; modelling; and decision-making. The mathematical models defining the main directions of the development of the theory of the firm are presented. The mathematical model for the development of the economy of the firm, in the form of a vector problem in mathematical programming (VPMP), is also constructed. The vector criterion of such a task represents a set of indexes characterizing the purposes of the development of the firm, with standard restrictions for resources. Further advanced is the numerical realization of the firm's development, in which the organization of work is presented in cluster form.

The second chapter deals with issues related to modelling and decision-making in market systems.

A mathematical market model is constructed in the form of a vector problem in mathematical programming, in which a balance between supply and demand is achieved and the focus of each participant on the market is considered. Information technology for model operation and the prediction of a market's development are presented along with ways of solving the practical (numerical) problems of a market's model operation.

The third chapter deals with issues related to modelling, forecasting and decision-making in the region. Leontief's model is the cornerstone here. First, it takes into account the purposes of each industry in the region and "expenses-release" is completed. Secondly, it takes into account any investment in the region's development. Thirdly, it looks at the dynamics of regional economic development. From here, an economic model of regional development is created in the form of a vector problem in mathematical programming. Its realization is demonstrated with a digital model of the region.

Keywords: theory of the firm, mathematical model of the firm, theory of the market, mathematical model of the region, vector optimization, methods for solving problems of vector optimization.

CHAPTER 8

THE THEORY OF THE FIRM
MODELLING AND MANAGEMENT DECISION-MAKING IN THE COMPANY

8.1. Introduction to the theory of the firm

The firm is the main economic object (subsystem) defining the social and economic development of municipality, the region and the state in general, as well as being the object on which the state's regional and market regulation of its economy is directed [61]. Therefore, much attention is paid in Russia and abroad — at both national and regional levels — to problems within the theory of the firm. In the West, some theories of the firm proceed from an ideology related to modern social and economic doctrines [62]. All of these focus on economic activity and the behaviour of the firm in a projected future period [63]:

• *The neoclassical*, or Marzhinalistsky theory, was considered useful for comparing one benefit to another; and the price of supply and demand was defined. In general, all schools of neoclassics differ in searching for methods of optimization with limited resources. The brightest representatives of the Lausanne School are Leon Walras [64] and Vilfredo Pareto [1]. Representatives of the Anglo-American School are Paul Samuelsson [65] and Alfred Marshall [66], who developed mathematical interpretations of economic processes.

• *The neoinstitutional* theory explains the existence of a variety of business enterprises, and includes problems around motivation for work, limits in the growth of a firm, its structure, questions of the firm's organization, problems with control and planning, and paying attention to the development of a firm's decision-making factors. This direction is rather widely presented in the discipline and known as the "theory of the organization." The most famous exponent of this theory is Ronald Coase, who received the Nobel Prize in Economic Sciences in 1991 [67]. For the first time within neoinstitutionalism, and using comparative research, he proved the contract nature of the firm, explained the concept of transaction expenses, and united the problem of the organization with an analysis of expenses.

• *Behaviouristic theory* studies the psychological aspects of consumer behaviour in terms of motivations and preferences in the choice and purchase of goods. Herbert Simon, who received the Nobel Prize in Economics in 1978 [68], continued the development of behaviourism in relation to economic processes. The essence of its concept is that the structure of an organization and its adoption of intra-organizational decisions are considered from the point of view of cooperative group behaviour. This further developed the concept of decision-making at a microeconomic level [69].

Current theoretical research into "the firm" is based on mathematical models predicting the behaviour of the firm in a projected future period. The widest circulation of this has been gained via models that proceed from criterion of maximizing, and the principles of decision-making in the firm are based on the limit analysis. Models which use criteria constructed on maximizing sales volume and growth belong to an alternative. The management theory of the firm is also relevant, as it is defined by the economic behaviour within the firm of those who are managers but not owners of the firm, and whose purpose is to maximize sales volume. The brightest exponent of this direction is the famous economist, J. Gelbreit, who was awarded the Lomonosov Gold Medal in 1993 [70].

In analysing production activity, J. Gelbreit emphasized the need for coherence in the purposes of society, the firm, and the personality. The Japanese model, which focuses on maximizing added-value [71], originates from this mode of thought. This set of mathematical models states that, separately, each of them represents situations which arise in practical terms during a firm's adoption of administrative decisions. The new directions connected with managing the development of a firm result in an economic theory of plurality of management purpose, which recognizes that a firm may have more than one purpose (e.g., profit, sales volume, growth, etc.), and that the set is worth more in total [72].

The purpose of this work is to analyse modern approaches to the theory of the firm and the creation of mathematical models defining the functioning of the firm within this approach. This work is devoted to research and development in this direction, taking into account the changing structure of the firm.

The mathematical model of the development of the firm unites the specified approaches which form the cornerstone of the problem of vector linear programming. The set (vector) criteria of such a task reflect the purposes and functioning of the firm in society, as well as solving practical tasks that arise when forecasting and planning developments in the organizational system, whether that be the firm, the region, or the state.

For the realization of a work goal (divided into two parts: the theoretical and the practical), analysis, modelling and forecasting developments in the firm are considered and solved:

- An analysis is carried out of the current state, via mathematical models, which define the main economic characteristics investigated in the theory of firm [62-75].
- On the basis of this analysis, a mathematical model is constructed (in the form of a vector problem in linear programming) of a production plan for the firm's economic development. The vector criterion of such a task represents a set of indicators characterizing the purpose of the firm's development, with standard restrictions on resources (materials, labour, capacity, etc.) [76-82]. To solve VPLP, the methods based on the normalization of criteria and the principle of a guaranteed result [11, 15 and 17] are used.
- An organizational scheme of management is created, along with a mathematical model of the firm's general control system [76-78].
- Decision-making techniques are shown, regarding problems within the modelling of the firm's development [78-79].
- A mathematical model is constructed to form a strategic plan, within which the development of the firm is shown dynamically, with some years taking into account extensive and intensive factors that increase the innovative activities of the enterprise [78-79].
- An algorithm for modelling the strategic plan is presented [79].
- It is demonstrated that, because of the resulting decision regarding the annual and strategic development plan of the firm, we achieve optimum output, according to the sales which most closely correspond to the chosen set of indicators [80-82].

The technology for modelling the firm's management needs to be discrete and so, to dynamically model in the form of a VPLP, a practical example is used regarding the formation of the firm's strategic development plan.

8.2. Analysis of the main economic theories of the firm and the creation of a multi-criteria model

8.2.1. Characteristics of the theory of the firm

The theory of the firm here refers to the firm's behaviour under various conditions. For example, the principles and motives of making decisions about prices, production, securities, investments, organizational structure, the acquisition of other firms, and mergers, etc. [62].

The process of forming the theory of the firm as a scientific theory, generally speaking, passes a number of interconnected stages at which certain problems are considered and solved:

1) The systematization of elements considered to show knowledge of reality. At this stage, there is supervision over existing processes, and an attempt to analyse these, using methods acquired from other sciences.

2) Definitions of the reason – consequences of and functional dependences between these elements, including the formalization and systematization of observed processes, drawing up their typology.

3) Detection of regularities, laws and tendencies in economic reality. This includes the development of applied scientific bases for the analysis and synthesis of observed processes; generalization of knowledge; and the creation of theoretical scientific fundamentals (e.g., principles, dependences, laws and regularities).

4) Development of a system of recommendations for the economic behaviour of an individual, the firm and the state. This requires the accumulation of statistical data on efficiency in the assumed methodology, and carrying out any necessary adjustment within it, as well as the creation of a methodology of the research process for the set typology [72, 74].

The first two stages of the systematization of the elements considered, show knowledge of the processes already present in the firm, and attempt to analyse them using known methods, as well as define their reasons. These are rather widely analysed in foreign reference works which proceed from an ideology of modern social and economic doctrines [63 - 71]. Therefore, we will pay most attention to the third and fourth stages.

The detection of regularities, laws and tendencies shown in the management of the firm is carried out by accumulating statistical data on th methods of management which allow the transference of an economic system from one state to another. Techniques for building and adopting administrative decisions are developed: planning, control, systems of communication and the motivations of administrative activity. On their basis, the management laws and principles that allow the methods and style of a firm to emerge are formed. The listed parties of management form a methodological basis from which rules and recommendations for practical activities can be created and followed by the heads and governing bodies of the firm.

For the development of systems to assess the economic behaviour of a firm in society and its purposes, tasks and management, various mathematical models are used. The following are currently used in theoretical research: maximizing profit, maximizing sales volume, maximizing "body height", administrative behaviour, and the Japanese model focused on maximizing added value [71]. These models and the research into them are presented in the following section.

8.2.2. Characteristics of mathematical models used in the theory of the firm

The traditional theory of the firm recognizes that the behaviour of the firm is defined by its only desire: to maximize profit. However, the emergence of the *model of maximizing* (***Model 1***) provides a simplified and abstract option. In the real world, though, a number of difficulties limit its adequacy. This is because, apart from an absence of comprehensive information, such a model demands that the firm precisely predicts the size and distribution of its future stream of profits. This is, at best, difficult and, at worst, impossible. Added to this, there are legal, ethical and social restrictions which place limits on the capacity to increase profit [71].

Maximizing profit becomes possible with equality between limiting expenses and income. Calculations of expense limits and income are difficult and there is no reliable and sufficient information about the market, as demand creates elasticity in both prices and income. It is therefore almost impossible to foresee the actions of a firm's competitors and to estimate the consequences of these activities. This means it is not possible to consider the traditional theory as one which is adequate to explain the behaviour of the firm in the best possible way. As a result, there are alternative theories to explain a firm's behaviour [72].

The model of maximizing sales volume (***Model 2***) is the most widely known alternative model for maximizing profit. It is straightforward to understand and is supported by attractive real-life examples. Empirical tests, however, don't confirm the hypothesis of maximizing sales [12]. The model of maximizing sales is made up of two parts, with the criteria of maximizing sales through a restriction on resources (***Model 2a***), and by minimizing the prime cost through restrictions on a number of economic indicators (***Model 2b***).

The model of maximizing growth (***Model 3***) is characterized by a strategy which has continuous growth as its cornerstone – an ongoing increase in production and sales over the long-term. Growth has to be financed by depreciation charges, assets, or loans [71]. This model has been used for several years.

The managerial theory of the firm claims that the economic behaviour of a firm is defined not by its owners, but by its managers, and that their purpose is to maximize sales volume. It is explained by a manager's direct dependence on their salary and the additional benefits they receive from trade revenue.

The model of administrative behaviour (***Model 4***) includes the model of administrative benefit, the model of administrative prudence, and the agency model.

The model of maximizing added value (***Model 5***), which is a Japanese model, is defined by the added value being calculated as the difference between a company's sales figures during a certain time period and the financial cost of goods acquired from external suppliers. Therefore, added value includes work, management, capital, and the costs of profit. Using such an approach, each worker and shareholder of the firm knows that, irrespective of economic conditions, priority has to be given to constant investment into capacity, equipment, research and development, and the development of the market. If the need arises to reduce remuneration to workers, the first course of action would be to reduce the salaries of senior administrative personnel. In such a way, Japanese auto-makers and other companies, regardless of economic conditions, seek to maximize added value year after year [71].

The emergence of such a set of models in itself suggests that each of them inadequately reflects the real situations which arise in the practice of managing a firm.

The economic theory of the plurality of purposes recognizes that the firm has a set of purposes (profit, sales volume, growth etc.). Today's firms include difficult corporate systems in which there are hierarchies of subjects which management have to try and make correspond to hierarchies of interests and purposes. The main concerns of those at the top of management are to increase the prestige of the firm, to improve the economic indicators which show the functioning of the company, and to provide stability. The main concern of a firm's shareholders is to raise high dividends. The primary concerns of a firm's managers are ensuring the furthering of their own careers, raising the company's social status, and creating income growth. The main concerns of hired workers in a firm are high salaries, good working conditions, professional development, and professional growth etc. [72]. This forms the basis for the creation of a mathematical model and modelling using the example of the annual, long-term (strategic) development plan of the firm.

8.2.3. The creation of a mathematical model from the firm's production development plan

As stated above, there are some alternative mathematical behaviour models of firms. We will unite the purposes of all models in the form of a criteria vector and consider the restrictions of each model. We will present a criteria vector and restrictions in the form of a mathematical model which represents a vector problem in mathematical programming.

The creation of a mathematical model of the annual (strategic) plan of a firm assumes the following formation: a vector of variables, a criteria vector (purposes) and restrictions imposed on the functioning of the firm [76, 77 and 78].

Vector of variables. Let $X = \{x_j(t), j = \overline{1, N}\}$ be a vector of variables for which every component is determined by $j \in \boldsymbol{N}$ and has the appearance and volume of $x_j(t)$ products, which are planned to be included in production during the planned year of $t \in \boldsymbol{T}$, with N as a set of indexes of types (nomenclature) of products, work, and services. Restrictions of u_j and $j \in N$ are imposed on the variables $x_j(t), j \in N$ — they determine the probable volume of the production of j-th of a kind. The sizes of u_j and $j \in N$ are determined through research into a commodity's market, which can be carried out by the firm, i.e. $x_j(t) \leq u_j(t), j = \overline{1, N}$.

The vector criteria define the purposes and functioning of the firm.

Production carried out in the firm is characterized by a set of $\boldsymbol{K}$ technical-economic indicators. We will designate functional dependence of any indicator of $k \in \boldsymbol{K}$ on the output of $X(t)$ through $f_k(X(t))$, on the assumption that such functional dependence exists. We assume that functional dependence regarding the $f_k(X(t))$ criterion is linear, i.e.,

$\forall k \in \boldsymbol{K}, f_k(X(t)) = \sum_{j=1}^{N} c_j^k x_j(t),$

where c_j^k is the k-th size of the indicator characterizing the unit of j-th production type, $j \in \boldsymbol{N}$.

In general, we will present everything that is an indicator in the form of vector functions:

$$F(\mathrm{X}(t)) = \{f_k(X(t)) = \sum_{j=1}^{N} c_j^k x_j(t), k = \overline{1, K}\} \quad (8.2.1)$$

From all sets of indicators of $\boldsymbol{K}$ we will allocate three subsets of indicators.

The first subset of the *criteria* of $\boldsymbol{K_1}$, $F_1(X(t))\subset F(X(t))$ depends on the organizational structure of the enterprises (the neoinstitutional theory). We consider a two-level control system of the firm. This represents a merger of enterprises which remain legally independent, but from which economic independence is partly transferred to the overall general management [75, 76]. There are various forms of organizing the management of enterprise mergers. These can be constructed as follows:

- By the "head of the firm" principle, according to which the heads of major offices form the management.
- By the holding principle, in which the management (board) can consist of the chairman, the board members responsible for financial activity, and members of the supervisory board. Such organization of production has found broad application for major companies in the Russian Federation.

The structure of an economic two-level control system of the firm, with information and administrative streams, is represented in Fig. 8.1.

Communication with a subsystem. Higher level.

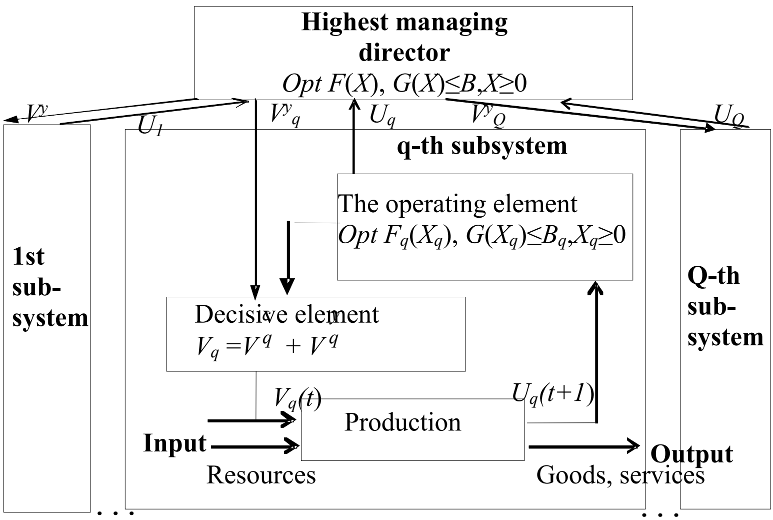

Fig. 8.1. The two-level hierarchical control system in a firm.

The two-level control system of the firm consists of one highest managing director of the *subsystem* (HS) and ***Q*** of the *local* operating *subsystems* (LS), subordinate by hierarchy. LS is the operated process (production). We will separately consider local structures and highest managing directors of a subsystem. We will establish interrelation between "entrance" and "exit" to LS. In the structure of LS, we will allocate three elements: production, the operating element, and a decisive element. We will show them in the example of q-th LS in Fig. 8.1.

Production. The functioning of LS depends on resources which arrive from the environment (for example, financial resources, including from the highest managing director of a subsystem). Resources in the course of production will be transformed into goods and services. Volumes of goods and services are characterized by a set of technical and economic indicators. U_q vector defines the nomenclature, volumes and technical and economic indicators at the exit q-oh of LS. The vector of U_q is transferred to the operating LS element and the highest operating subsystem.

The known operating elements are: purposes; product range, which can let out LP; resource costs of the finished product; potential opportunities in acquisition; and the need in the market for the goods on the basis of which LS forms its own vector of management V_q^c, i.e., the vector of V_q^c is, in essence, the information model of the q-th division. The vector of V_q^c is reported to the LS decisive element for a final decision.

The decisive element is intended to associate with its own vector of management V_q^c and q-th components of a vector of the management V_q^y received from the highest managing director of a subsystem. We assume that the structure of both vectors is identical and can therefore be united (to put):

$$V_q = V_q^c + V_q^y$$

which is the resulting management vector. V_q (at entrance) serves as the final operating solution for the production q-th of the enterprise.

It is supposed that the firm consists of Q – a set of the separate enterprises (divisions), which each have development purposes that functionally depend on output $f_q(X(t))$, $q \in Q$. The enterprise is presented with a set of criteria of K_q, $q \in Q$:

$$f_q(X(t)) = \{f_{kq}(X(t)),\ k = \overline{1, K_q},\ q \in Q\}.$$

If Q=1, this is the standard definition of the firm.

In total, the purposes of the separate enterprises are presented as a vector function:

$$F_1(X(t)) = \{f_q(X(t)),\ q = \overline{1, K_1}\},\ K_1 = Q, K_1 \subset K.$$

• *The second subset of the **K_2** criteria* defines the purposes of the firm in general, the known highest operating subsystem for which is the data on each LS; the purposes of their functioning; the product range which LS can allow; the resource costs of products; LS potential; the purposes of the functioning of all firms in general; and global restrictions on resources such as finance.

This information is processed by VS and the operating vector is developed as a result. The purposes of the separate enterprises are presented as a vector function:

$V^y = \{V_1^y, \ldots, V_q^y, \ldots, V_Q^y\}$ –

This is a coordination vector which, for each LP, contains the nomenclature, volumes, and technical and economic indicators of products.

$F_2(X(t)) = \{f_k(X(t)),\ k = \overline{1, K_2}\},\ \boldsymbol{K_2} \subset \boldsymbol{K}.$

The vector of $F_2(X(t))$ includes the sales volumes of the finished product, the profits, and added value, etc.

It is desirable to maximize these indicators.

• *The third subset of **K_3*** which is desirable for minimizing:

$F_3(X(t)) = \{f_k(X(t)),\ k = \overline{1, K_3}\}$ and $\boldsymbol{K_3} \subset \boldsymbol{K}$ are the indicators connected with the minimization of the material, labour input, the capacity to define total prime cost, and products.

Indicators of $\boldsymbol{K_2}$ and $\boldsymbol{K_3}$ submit system characteristics of the firm and define its interrelation with society.

$\boldsymbol{K_1} \cup \boldsymbol{K_2} \cup \boldsymbol{K_3} = \boldsymbol{K}$ — a set of indexes of indicators (criteria) of the development of the firm in general.

Restrictions. These are considered when developing the plan and are connected to:

- Resources, i.e., the enterprise’s capacity, labour, and material resources.
- The minimum required planned indicators.

Restrictions on resources. We assume a linear dependence of resource expenses on the volume of the finished goods $X = \{x_j(t), j = \overline{1, N}\}$ with restrictions:

$$\sum_{j=1}^{N} a_{ij}(t) x_j(t) \le b_i(t),\ i = \overline{1, M}, \quad (8.2.2)$$

where $a_{ij}(t), i = \overline{1, M}, j = \overline{1, N}$ - is the quantity of an *i*-th resource necessary for the production of the *j*-th unit product type.

The set of indexes of resources $\boldsymbol{M}$ includes:

- A set of the $\boldsymbol{M_{mat}} \subset \boldsymbol{M}$ resources which characterize materials and the semi-finished products, etc., which are used in production.
- A set of manpower (specialties) of $\boldsymbol{M_{tr}} \subset \boldsymbol{M}$ participating in production.
- A set of the accumulated resources (capacity) of $\boldsymbol{M_f} \subset \boldsymbol{M}$.

Similarly (2) we will present expenses on i-mu for a resource of the q-th of the division:

$$\sum_{j=1}^{N_q} a_{ij}^q x_j(t) \leq b_i^q(t), i = \overline{1, M_q}\,, q = \overline{1, Q},$$

where $b_i^q(t), i = \overline{1, M_q}\,, q = \overline{1, Q}$ — is the size of the i-th resource which is available in the q-th division of the enterprise for the planned period. $\boldsymbol{M_q}$ — is a set of types of resources which are used in production in the q-th division.

The restrictions connected with planned indicators:

$$\sum_{j=1}^{N} c_j^k x_j(t) \geq b_k(t), k \in K, \qquad (8.2.3)$$

where c_i is the size of the j-th unit for the economic indicator characterizing production; and b_k is the minimum size of a planned indicator which should reach the firm.

When creating models, we will specify a number of indicators used for criteria and restrictions.

The costs of production in a unit are subdivided into variables and constants depending on their participation in production.

Variable expenses depend on output:

$a_{ij}(t), j = \overline{1, N}, i = \overline{1, M}$ - the resource i-th costs of the j-th unit for a production type (norm); I; $\boldsymbol{M}$ is an index and a set of all types of resources, such as material and labour (i.e., the variable expenses that change in proportion to output and which are used in the process of all types of production;

$$G_i(X) = \sum_{j=1}^{N} a_{ij}(t) x_j(t) \leq b_i(t), i = \overline{1, M}$$

is the resource i-th costs of all types of production.

For the prime cost of a unit of production, using understood expenses, we will present the sums of the costs of variable expenses in the following form:

$$a_j^p = \sum_{i=1}^{M_{mat}} p_i a_{ij}^{mat} + \sum_{i=1}^{M_{tr}} p_i a_{ij}^{tr} + \sum_{i=1}^{M_f} p_i a_{ij}^{f}, j = \overline{1, N}, \qquad (8.2.4)$$

where $a_j^{mat}, a_j^{tr}, a_j^{f}$, p_i is the cost of a unit of production and the material, labour and accumulated resources respectively; $A^p(t) = a_j^p(t), j = \overline{1, N}\}$ is a vector of the prime cost of planned production in units of all types.

Constant expenses don't depend on output; they are paid off from product units - a_j^{nak} (depreciation charges, administrative and managerial expenses, maintenance costs of buildings and equipment, etc.). In general, the planned full prime cost of a unit of production is defined as the sum of the prime cost of production and overhead costs:

$$a_j = a_j^p + \ a_j^{nak}, j = \overline{1,N} \text{-} \qquad (8.2.5)$$

This is a basis for the formation of the market value of production.

Similarly, the costs of production per piece on goods which are like those produced by a competitor company are compared and defined as far as is possible.

The price of a unit of production for the p_j, j-th type follows from market research, or assumes the calculation of a price, taking into account the price policy in relation to each concrete market (segment) and the global policy of the firm. An important role is therefore played by the methods for calculating the settlement price from the prime cost of $a_j, j = \overline{1,N}$.

Profit: the gross unit profit is defined as the difference between the cost of the j-th production type of p_j and variable expenses

$$\pi_j^{val} = p_j - a_j^p, j = \overline{1,N};$$

the profit on product sales in the firm is, in general, equal to

$$\pi = f_k\big(X(t)\big) = \sum_{j=1}^{N} \pi_j x_j(t)\,, k \in \boldsymbol{K},$$

where the number of criterion is as $k \in \boldsymbol{K}$.

Profit which is subject to distribution: $\pi^{pd} = \pi - c^{div}$, where c^{div} - the total dividends which are assumed to be delivered in the planned year.

The added value is defined in a unit of production as the difference between the production cost and the material input of the j-th type:

$$p_j^{dob} = p_j - a_j^{mat}, j = \overline{1,N} \qquad (8.2.6)$$

where $a_j^{mat} = \sum_{i=1}^{M_{mat}} p_i a_{ij}, j = \overline{1,N}$ is the cost of material input in for the j-th unit production type from external producers.

Using the calculated indicators (8.2.1)-(8.2.6) we will construct the above-mentioned theoretical models:

• maximizing profit (***Model 1***) - to define

$$max\, f(X(t)) \equiv \sum_{j=1}^{N} \pi_j x_j(t), \qquad (8.2.7)$$

with restrictions $\sum_{j=1}^{N} a_{ij}(t) x_j(t) \le b_i(t), i = \overline{1,M}$, (8.2.8)

$$0 \le x_j(t) \le u_j(t)\,, j = \overline{1,N}; \qquad (8.2.9)$$

• maximizing sales volume (***Model 2a***) - to define

$$max\, f(X(t)) \equiv \sum_{j=1}^{N} p_j x_j(t), \qquad (8.2.10)$$

with restrictions $\sum_{j=1}^{N} a_{ij}(t) x_j(t) \le b_i(t), i = \overline{1,M}$, (8.2.11)

$$0 \le x_j(t) \le u_j(t)\,, j = \overline{1,N}; \qquad (8.2.12)$$

• minimization of total expenses (***Model 2b***) - to define

$$min\, f(X(t)) \equiv \sum_{i=1}^{M} c_i \sum_{j=1}^{N} a_{ij}(t) x_j(t), \qquad (8.2.13)$$

with restrictions $\sum_{j=1}^{N} c_j^k x_j(t) \geq b_k(t), k \in K,$ (8.2.14)

$0 \leq x_j(t) \leq u_j(t), j = \overline{1,N},$ (8.2.15)

where c_j^k is the economic indicator characterizing the j-th unit production type; b_k is the set economic indicator characterizing such a volume of production at which b_k has to be executed (≥);

• *maximizing growth (**Model 3**) – to define*

$max\ f(X(t)) \equiv \sum_{j=1}^{N} \pi_j x_j(t),$ (8.2.16)

at restrictions $\sum_{j=1}^{N} a_{ij}(t) x_j(t) \leq b_i(t) + \Delta b_i(t + \Delta t), i = \overline{1,M},$(8.2.17)

$0 \leq x_j(t) \leq u_j(t), j = \overline{1,N},$ (8.2.18)

where $\Delta b_i(t+\Delta t)$ is a gain of resource volume in the planned period Δt which is created due to depreciation charges or loans;

• *administrative behaviour (**Model 4**) – to define*

$max\ f(X(t)) \equiv \sum_{j=1}^{N} p_j^z x_j(t),$ (8.2.19)

with restrictions $\sum_{j=1}^{N} a_{ij}(t) x_j(t) \leq b_i(t), i = \overline{1,M},$ (8.2.20)

$0 \leq x_j(t) \leq u_j(t), j = \overline{1,N},$ (8.2.21)

where p_j^z is the economic indicator characterizing salary volume from production and the unit sales of a j-th production type;

• *model of maximizing added value (**Model 5**) – Japanese model* – to define

$$max\ f(X(t)) \equiv \sum_{j=1}^{N} p_j^{dob} x_j(t), \quad (8.2.22)$$

$with\ restrictions\ \sum_{j=1}^{N} a_{ij}(t) x_j(t) \leq b_i(t), i = \overline{1,M},$ (8.2.23)

$0 \leq x_j(t) \leq u_j(t), j = \overline{1,N},$ (8.2.24)

where p_j^{dob} is the the economic indicator (8.2.6) characterizing the volume of the added-value from the production and sale of the j-th unit production type.

All presented models are used in the creation of a common mathematical model of the firm, which is shown in the following section.

8.2.4. The mathematical model of the firm constructed on the basis of vector optimization

The economic theory of plurality of purposes assumes that the listed goals (criteria) of models 1-5 exist and have to be considered by the management of the firm. This purposefulness of models 1-5 will be presented in the mathematical model of the firm in the form of a vector problem in linear programming [76, 77 and 78]:

$opt\ F\big(X(t)\big) = \{max\ F_1\big(X(t)\big) =$

$\Big\{max\ f_q\big(X(t)\big) = \Big\{maxf_{kq}\big(X(t)\big) \equiv \sum_{j=1}^{N_q} c_j^k x_j(t)\,, k = \overline{1, K_q}\Big\}, q = \overline{1, Q}\Big\}\},$ (8.2.25)

$max\ F_2\big(X(t)\big) = \big\{maxf_k\big(X(t)\big) \equiv \sum_{j=1}^{N} c_j^k x_j(t)\,, k = \overline{1, K_2}\big\},$

$min\ F_3\big(X(t)\big) = \big\{minf_k\big(X(t)\big) \equiv \sum_{i=1}^{M} c_i \sum_{j=1}^{N} a_{ij}(t)x_j(t)\,, k = \overline{1, K_3}\big\},$ (8.2.26)

with restrictions $\sum_{j=1}^{N} a_{ij}(t)x_j(t) \le b_i(t), i = \overline{1, M},$ (8.2.27)

$\sum_{j=1}^{N_q} a_{ij}^q x_j(t) \le b_i^q(t), i = \overline{1, M_q}\,, q = \overline{1, Q},$ (8.2.28)

$\sum_{j=1}^{N} c_j^k x_j(t) \ge b_k(t), k \in K,$ (8.2.29)

$0 \le x_j(t) \le u_j(t)\,, j = \overline{1, N},$ (8.2.30)

where $F(X(t))$ is the vector criterion in which $\boldsymbol{K_1}$ is a subset of criteria of the firm's divisions, $\overline{1, K_q}, q = \overline{1, Q}$, $\boldsymbol{K_1}$=$\boldsymbol{Q}$;

$\boldsymbol{K_2}$ is a subset of criteria in which every component is maximized (sales volumes of the finished product, profits, added value, etc.);

$\boldsymbol{K_3}$ minimizes (these are the indicators connected by the prime cost of products);

$\boldsymbol{K_2}$ and $\boldsymbol{K_3}$ are the criteria systems characterizing the activity of the firm in general. The criteria vector of $F(X(t))$, in total, reflects the purposes of all models, from (8.2.7) model of profit to (8.2.22) model of maximizing added value;

$X = \big\{x_j(t), j = \overline{1, N}\big\}$ is a vector of variables for which every component is defined as a quantity of j-th product type included in the plan;

c_j^k is an economic indicator of k-th, of a type of $k = \overline{1, K_1}$, j-th for the type of production characterizing the unit.

Restrictions (8.2.27)-(8.2.30), in total, reflect the restrictions of all models, from profits (8.2.8)-(8.2.9) to models of maximizing with added value (8.2.23)-(8.2.24).

We will notice that there is a problem of definition:

$\forall q \in \boldsymbol{Q}, maxf_q(X(t)) = \{maxf_{kq}\big(X(t)\big) \equiv \sum_{j=1}^{N_q} c_j^k x_j(t)\,, k = \overline{1, K_q}\},$

with restrictions (8.2.27)-(8.2.30)

which is a model of a firm's separate divisions and represents a vector problem in linear programming.

To solve a vector problem in linear programming (8.2.25)-(8.2.30), methods based on the normalization of criteria and the principle of a guaranteed result are used. These are presented in the first volume [11, 15 and 17].

8.2.5. The general scheme of a firm's management and its place in the mathematical model

In an organizational system (including the firm, the market, a branch, a municipality, a region, and the state) there are two main functioning levels of activity: production and management. From the point of view of the system approach, we can consider these as the object and subject of management (see Fig. 8.2.).

Management represents the sequence of the operating impact of the subject on the object of management, and is intended to ensure the functioning of the organizational system in the set mode, according to the purposes of its existence and development. The operating influence is a result of management [17].

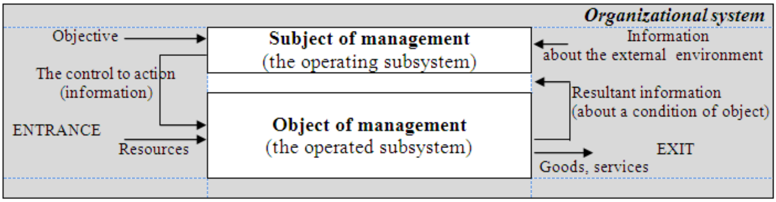

Fig. 8.2. General scheme showing the relationship between the subject and object of management in an organizational system.

The control to action represents information about how and when the object of management should take place. This is often called the command information. Management is considered as a continuous process of development of control actions (commands), carried out over time, in the management process.

The management process is the purposeful activities of the subject of management, in order to coordinate the joint functioning and development of all divisions of the object of management, which are dynamically changing in space and time. The management process is subdivided into separate subprocesses: forecasting, planning, decision-making, accounts, control, analysis and regulation.[1] These subprocesses have been given the names of the functions of management and are used by an operating system at various stages and levels of management.

[1] A. Fayol allocated five elements of management: anticipation, the organizations, stewardship, coordination, and control. These are called "further functions of management" [76].

All functions of management in concrete organizational systems represent a sequence of interconnected actions and operations, and are shown in Fig. 8.3. In Fig. 8.2, these functions are presented in one block: the subject of management. In Fig. 8.3, the logical interrelation between the functions of the subject of management (Fig. 8.3b) and the functions of the object of management (Fig. 8.3a) is also shown.

In Fig. 8.3b, two contours of management are allocated: planning and control. These help the main functions of the management process to be realized.

The contour of planning is presented through a sequence of functions in forecasting, planning, decision-making (according to the plan) and regulations. These are the contours of direct control.

The contour of control is presented through a sequence of functions in accounts, control, analysis, decision-making (on control) and regulations. This is a feedback contour in a chain of management [17].

The object of management (Fig. 8.3a) is presented through production functions which include:

- Production material support.
- Direct production (production shops, technology, the condition of fixed assets, wear and tear, and restoration);
- The realization and release of goods and services.

The realization of *the objects* set by management at various levels in a firm is carried out through planning functions.

The function of planning includes various stages (see figs. 3b→3c).

Stage 1. *Statement of an economical problem*. At this stage the following is carried out:

- The preparation of statistical information, including forming the main economic indicators that will be used for future planning, as well as reporting data for the previous and current years.
- Analysis of reporting and statistical information for the main economic indicators concerning the implementation of the plan for the previous period, and the forecast on the basis of the firm's economic development at its current stage of functioning (Fig. 3a).
- Market research.

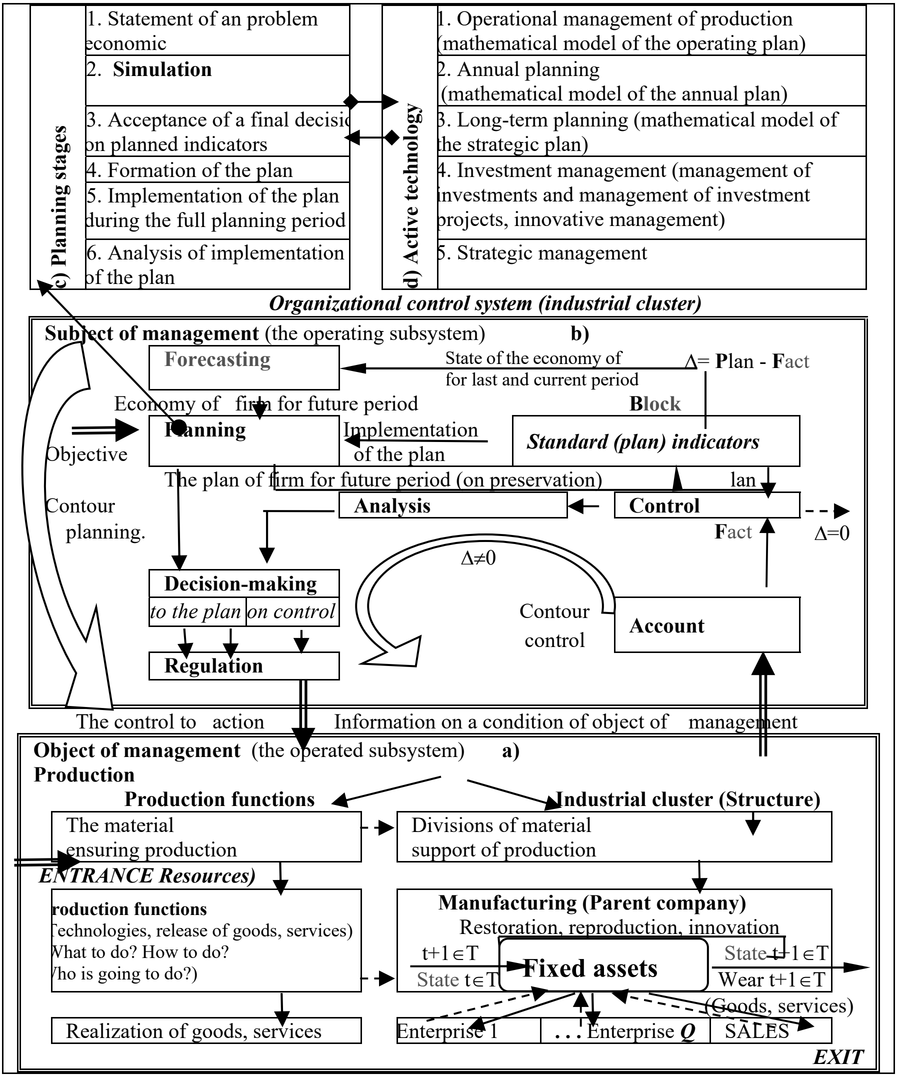

Fig. 8.3. Functions (subprocesses) of management and production: a) object of management; b) subject of management; c) planning stages; d) management (active) technology.