# Formal Methods in Architecture and Urbanism, Volume 2

# Formal Methods in Architecture and Urbanism, Volume 2

Edited by

Catarina Ruivo, David Leite Viana, Franklim Morais and Jorge Vieira Vaz

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#### INTRODUCTION

## CATARINA RUIVO, DAVID LEITE VIANA, FRANKLIM MORAIS, AND JORGE VIEIRA VAZ

The International Symposium Formal Methods in Architecture is a biennial scientific meeting that has taken place in Portugal since 2011. The editors are members of their committees. Most of the papers of the 3rd edition of the Symposium (2015) were published in a book, *Formal Methods in Architecture and Urbanism*, by Cambridge Scholars Publishing.

This new volume includes a selection of papers presented to the 4th edition of the Symposium (2018). The editors would like to add a few preliminary words.

#### **Formal Methods in Architecture**

The main purpose of the symposia is a contribution to the debate about the application, in the disciplines of Architecture and Urbanism, of new formal methods—new methodological advances based on tools coming from Mathematics. From the millennial geometry to current shape grammars, several formal approaches to Architecture and Urbanism are presented, with their different points of view, different fields of application, and different grades of abstraction and formalization. The aim is to look at the potentials and purposes of these formal methods, both those on the horizon as well as those already accomplished, and both their successes and their problems. The intention is to promote the use of formal methods in the creation of new explicit languages for problem-solving in Architecture. These problems range from representation, to theory, critique, production, and communication, etc., never ceasing to see Architecture and Urbanism as technological activities as well as artistic ones.

The main fields of interest are the collection and low-level semantic organization of information (several methods of tracking and mapping—video; GPS, Wi-Fi; ISP; cellular phones; beacons; IoT-Internet of Things; GIS-Geographic Information Systems; BIM-Building Information Model/VDC-

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Virtual Design and Construction); syntactically and semantically advanced formal languages (ontologies for the built environment; machine learning processes; shape grammars); formal theories (Space Syntax; SCAVA-Space Configuration, Accessibility and Visibility Analysis; cellular automata; agent-based design); project representation, visualization, and interaction (CAD-Computer Aided Design/BIM-Building Information Modelling; virtual reality; augmented reality; spatial augmented reality; human-computer interaction); low, medium, and high-level architectural design automation (CAD-Computer Aided Design/BIM/IFC-Industry Foundation Classes: parametric design; processing); advanced automated architectural design (shape grammars computer implementation; advanced reasoning artificial intelligence tools—heuristic and non-heuristic); building performance analysis (environmental analysis; multi-criteria analysis; flow and crowd analysis; biometric sensing of users); automated manufacturing—CAM (subtraction techniques; adding techniques; tessellation techniques); and the active management of the built environment (participatory urbanism; smart buildings; smart cities).

The more historically established areas for the application of mathematical sciences, such as traditional geometries or mathematical developments connected to engineering, are left somewhat outside our focus, without forgetting, however, the deep connections between them. Some of these new methodologies have a level of development that requires the existence of established academic communities, with their own specialized forums. Our symposia, which are more than an attempt to deepen each specific field (which are also), are above all about finding points of convergence. This is not limited to a possibly interesting abstract integration of different areas of research but it is mainly concerned with advancing the multiple crosses between various methods, whose fertility has already been proven.

A dialogue with semi-formal and even informal methods in current use has also been stimulated as a way to deepen the discussion on the controversies of aesthetics and ideologies that surround the possibilities and reach of a formalization of Architecture and Art. Even some contributions on the application of formal methods in fields other than architecture, like literature, music, and the fine arts, have been useful for architectural practice. These considerations could lead to the conclusion that the work is based on discussions of generalities and emptiness, which is not the case. A high level of technical depth is required on every single contribution. Formal rigor and acuity are constant demands for all participants.

#### The Structure of this Book

This volume aggregates about three dozen papers based on the Symposium presentations. The scope is the built environment, but a wide range of scales is presented, from the decorative tile to the room, from the house, from the building to the urban scale, from the street, and from the square to the city.

This volume is divided into two parts.

Part I includes the chapters that deal with the construction of the formal representations of the built environment. They start from the collection of relevant information, go through the semantic organization of that information with increasingly elaborate levels of semantic abstraction, and finally lead to theories, as well as structured and formal mental constructions about the built environment (the forms) and their relations with both the natural world and the psychic and social world of human beings (the functions). These theories, with a greater or lesser degree of complexity and abstraction, make it possible to carry out assessments on the desirability of architectural or urban solutions.

#### Part I has five sections:

Part I.1 encompasses texts that preferably use SCAVA-Space Configuration, Accessibility, and Visibility Analysis (e.g., Space Syntax, isovists, visual graph analysis, or agent-based analysis) methods and theories in urban problems. Most intend to establish the interrelations between the configuration of urban spaces and the human activities that develop in them, as both (forms and functions) interact and condition each other. However, these analyses are also concerned with the semantics within the language of the forms/spaces themselves for purely configurational and/or aesthetic evaluations, as is shown in Chapter 1.

Part I.2 groups texts that are still dedicated to urban analysis but which use other methods, especially GIS and its additional analysis and evaluation tools. The use of GIS and SCAVA methodologies together are also shared, as some of the texts in Part I.1 indicate. In Part I.2, texts with less formal methodologies are also presented. Chapter 10 is a good example of a more classic semi-formal analysis that can establish a good dialogue with formal methods. Although not completely formalized, some concepts from classical theories are relevant to a deep analysis of architectural practice and may be the object of the creation of future formalisms.

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Part I.3 contains two texts with SCAVA methodologies applied to buildings, and Part I.4 does the same but in this case they are applied to landscape architecture.

Part I.5 contains two texts. One of them brings to our attention a very classic analysis of architectural theory, which contains many concepts that are also subject to formalization today. The other text makes a classic approach to a generic problem of urban studies but points out the subordinate lines in which it contemplates both more classic and more formal methods. Both texts serve to continue the discussion on the interaction dynamics between analysis and theory, either classical or formal, with the different forms that this interaction can take.

Part II deals with automatic design and project production methodologies. These formal methodologies for producing the built environment are much less developed than those related to the analysis of the built environment.

Part II.1 contains a single text, which is representative of part of the linguistic formalization effort. Any formal language needs very firm syntactic bases, so some studies have to be directed exclusively at the formal definition of very generic syntaxes and semantics. It is a work similar to that of mathematicians who provide languages for all sciences, although they are not tied to the practicality of their use.

Part II.2 brings us, mainly, generative languages that programmatically produce forms for environmental projects to be built in the future. Usually, many of these grammars contain exclusively syntactic production rules, so the forms generated are arbitrary and without any control from the point of view of their ability to interact with their exterior—the lives of human beings. Many of the chapters in Part II.2 overcome this problem. Those generative grammars use production rules that already take the knowledge that comes from the consolidated heuristics of architects into account. Grammars based on heuristic, cultural, aesthetic, and even social knowledge are presented. Some of the grammars are even built from the observation of reality and then used for future developments. Machine learning methods are used for this construction.

Part II.3 includes very interesting examples of the use of formal methods of production in an academic environment, with a view to training new architects to be sensitive to formal methods.

Part II.4 contains a single text that serves as an introduction to a dialogue with some trends of contemporary architecture which, although using semiformal methodologies, are similar to formal tools.

# 4<sup>TH</sup> SYMPOSIUM FORMAL METHODS IN ARCHITECTURE: CLOSING REMARKS

#### FRANKLIM MORAIS

#### **Honesty and Modesty**

During the course of the Symposium's work, two of the presentations shook our minds. Not because the ideas and positions were so different from the rest, but because they reached two core points of an ethical behaviour.

In the first, (1) one of the authors presented a study where a set of theses were proposed to explain the influence of the permeability of facades on urban vitality. It was an application of Space Syntax methodologies to some observed behaviours in a Brazilian city. But this is not our topic. What we are trying to relive is that, at some point in the speech, the reader presented the conclusions, and said, "this and that thesis confirmed; this and that thesis not confirmed". The three magic words were "thesis not confirmed". Many of us are used to a discourse on architecture where there is no research and no confrontation with reality. It only makes statements of principles, which are often ideological and/or normative. And yet there are those who first find the revealed truths and then the necessary ad hoc demonstrations to support them. Many proclaimed theses are clearly biased by ideological claims. It must be stressed that nothing moves us against ideology. There is no decision without some human appreciation and evaluation. The scientific commitment is only against the self-presentation of ideology as theory, and of norm as truth. The authors of this paper were clearly attached to some social, political, or cultural positions. Nevertheless, that did not prevent them from accepting partial defeat. They did not try to impose their wills on their results. They positioned themselves in the place of total honesty: the confrontation of their thesis with reality, through their theoretical method,

<sup>(1) &</sup>quot;Development of a Permeability Measure Between Private and Public Space", Patricia Alonso, Meta Berghauser Pont, and Luiz Amorim (Chapter 7).

corroborated some of them and those that were not confirmed had to be forsaken.

In the debate over another paper, (2) the author undervalued his own work, saying that it had not achieved a great level of conclusions. For some of us, who have been following his work for some time, this seemed somehow unfair. Actually, he has performed an overwhelming piece of work, with very detailed analysis on hundreds of squares, in order to obtain some conclusions on their configuration and behaviour. He had mathematically formalized several dozens of parameters, trying to get a formal definition and clustering of types of squares. At the end, the author was not that pleased. He just said that the work was not so good after all. But we could understand his modesty: despite all his work, he was still very far away from a semantically well founded "theory of the square". These same ethical positions globally and implicitly pervade all communications. Unlikely the traditional architectural papers, many of them used the current scientific template: they explicitly presented the underlying methodological base, they made experiments, their thesis was formally constructed, the results were discussed, and they usually presented the labour to be performed in the future to rectify the flaws of the present. We believe that the use of formal methodologies has a prominent role on these ethical (among others) achievements. Formalization implies making all the elements of the "speech act" completely explicit, implying not only the "speech" but also the process. Totally explicit knowledge needs the inclusion of the interpretative code to be used by other languages to make clear assumptions. The concepts of those structures are becoming increasingly diverse from those immediately tied to empirical perceptions of objects and those trying to search for essences of the reality far away from our senses. Nevertheless, their construction may be entirely mapped from the first assumptions to final results though all the algebraic movements of their construction. They can thereby be related to empirical data.

When someone presents a formal theory, they declare all the global assumptions (the code to interpret the theory, including the code for other people to understand the theory), all the terms, and all the rules of the generative grammar for creating new terms. Nothing can be hidden. One of the major tasks of a new theory is the explicit definition of its domain—the portion of reality it has to deal with. This means that each theory has a perfect understanding of what each theory is missing, which parts of reality

(2) "Linking Data Mining, Spatial Analysis and Algorithmic Design", João V. Lopes (not in this book).

-

it deliberately ignores, and its incompleteness. It never tries to be the answer to all problems. It knows that it is provisional and carries errors inside it, but it can also be improved in an endless process.

#### The Formalization of Art and Art Studies

I would like to stress a final topic: formalization (representing something in a formal language), which is not directed against Classic Studies. For example, in one other paper given at the Symposium, <sup>(3)</sup> the author brought us some old ideas in refreshed suits. The presentation focused on a semantic definition and organization of some now underrated concepts, such as grace, scale, decorum, order, and composition.

We might think we were moved in time and place to Quais Malaquais in Paris, in the late nineteenth century or the beginning of the following, where the lessons of the courses of Architecture brought us the Treatises of the Beaux Arts, from Guadet or, later, from Gromort and Gutton. For some, those treatises have already been thrown to the dustbin of History, aided by the numerous schools of "Arts and Crafts" and even more by the Vkhutemas and Bauhaus schools. This skirmish can be viewed from a broad perspective as an episode of the millennial conflict over rhetoric.

The position of Plato is well known: rhetoric is a good mean for the persuasion of the ignorant masses and not for the discovery of truth. Aristotle countered that it was good to study rhetoric because everyone could learn how he was deceived, but yet more relevant is the fact, forgotten by Plato, that even the mighty truth needs to materialize in some language to be revealed.

Modern speech act theory teaches us that sentences in natural speech are predominantly performative utterances. Semiotic studies do not say otherwise. So, even Plato's "Gorgias", where he painfully strikes rhetoric, is a rhetorical masterpiece. Even architectural modernism, whose discourse is quite far from rhetoric, could not avoid the use of architectural languages with their implicit rhetoric. Paraphrasing the mocking argument of someone

<sup>(3) &</sup>quot;Form and Meaning in Architecture and Urbanism: Principles of Quality", Javier Poyatos Sebatián (Chapter 16).

(Robert Venturi), Beaux Arts has made monuments to something; modernism makes monuments to itself.

The Treatises of the Beaux Arts carried a great flaw: their linguist tropes were presented as perennial norms, giving them the very bad reputation of academism. But linguistic studies could split semantic concepts from their values and could reuse what was a real advance in conceptual systematization. Many of the old concepts are now formalised and are not tied to specific values. Looking further back, the old treatises did not suffer from the same flaws of academism. Maybe what we are really trying to do is to translate Alberti's concinnitas into a formal language.

#### -PART I-

# REPRESENTATION OF THE BUILT ENVIRONMENT: FROM INFORMATION GATHERING TO THEORIES AND EVALUATION

# **I.1**

# SCAVA METHODOLOGIES IN URBAN STUDIES

#### CHAPTER ONE

# ALEXANDER'S THEORIES APPLIED TO URBAN DESIGN<sup>(1)</sup>

## ALICE RAUBER, ROMULO KRAFTA

#### Introduction

The relationship between urban design and science has always been highly debatable<sup>1-4</sup>. Using analytical methods in urban design has been a challenging question since they do not easily become part of the process.<sup>5</sup> <sup>6</sup>. Despite the difficulties, several authors claim that urban design should be more evidence-based, <sup>7–10</sup> especially because of the increasing availability of data for cities and analytical tools, which opens up the possibility of a science of cities. 11, 12 In such a context, it is worthy to review some of the Christopher Alexander's theories. He is an architectural theorist pioneer in trying to find better ways to design. Since the 1960s, he has been outlining a theory of built space design that seeks to overcome the architectural/urban production as something conceived purely in arbitrary and subjective terms. Although he is best known by his seminal work A Pattern Language, 13 the focus here relies on the theories presented in The Nature of Order. 14 and related work. 15, 16 In his recent work, 14-16 Alexander has proposed very instigating theories about the harmony and beauty seen in natural and manmade artefacts, although his concepts remain very hard to grasp and unlikely to practical applications. Here we try to bring such concepts closer to a science-based urban design development. First, we briefly introduce the main theories.

Wholeness is the central concept in recent Alexander's work. 14-16 It can be defined as a global structural character of a given configuration existing in space, both in natural and man-made things. Alexander also calls it a living

<sup>(1)</sup> This chapter was previously published in the Special Issue "Formalizing Urban Methodologies". *Urban Sci.* 2018, 2(3), 86; https://doi.org/10.3390/urbansci2030086.

structure because it emerges from an incremental process. He argues that everything has some degree of life. Of course, he refers to non-biological sense. His meaning for life is more related to coherence and harmony.

Wholeness structure is composed of primary entities called centres. Those centres support and intensify each other through the repeated occurrence of fifteen geometric properties: levels of scale, strong centres, boundaries, alternating repetition, positive space, good shape, local symmetries, deep interlock and ambiguity, contrast, gradients, roughness, echoes, the void, simplicity and inner calm, and not-separateness. Such properties describe how the centres interact with each other. Properties help to increase the coherence and the strength of any given centre and to generate new ones. Alexander believes that the fifteen properties play a major role in making the wholeness of a system, because of the recurrence of them in all the coherent systems observed by him through many decades. Wholeness is, therefore, a recursively defined structure composed of centres, which in turn are composed of other centres. The successive application of transformations, in other words, computations, based on the fifteen properties lead to the formation of living structures. Such a process is harmony-seeking or wholeness-extending oriented<sup>16</sup> since it seeks to preserve the previous structure. However, it only happens when transformations are based on the fifteen properties.<sup>16</sup>

As we can note, the idea of wholeness addresses the underlying structure of systems, namely its hidden quality. The existence of sub-structures—the centres and the sub-centres—addresses a scaling hierarchy. Such understanding helps to bring design process closer to a complex sciences approach. According to Alexander, <sup>15</sup> scientists who study biology and physical phenomena, for example, are passive in regard to the aspect of creation. The architects, on the other hand, are key proponents, whose project errors interfere in the lives of many people so they should be aware of the design process. Therefore, the creation of complex structures—which is the case of architecture and urbanism—should become an important scientific topic. Alexander suggests that aesthetic plays a key role in the coevolution of complex systems since the transformations that lead to the emergence of harmony and beauty in nature obey universal rules—the 15 properties. If it is true, such a process should be understood in order to be applied to the design process.

Finally, Alexander<sup>14-16</sup> argues that wholeness, namely the degree of life of a given configuration, is measurable since the properties can be objectively observed and described. However, the author himself admits that we still do

not have a mathematical language or a computational method to achieve this. According to Alexander<sup>14</sup> (pp.364-367), although the measurement of wholeness relies necessarily on the human observer, it is not just a cognition problem, but something objective that exists in space. The author admits that we need a more objective and mathematical way to support the task of measuring wholeness in spite of his deep concern for human intuition, especially when analysing complex artefacts such as cities and building. He proposes a new research agenda to operationalise the harmony-seeking process<sup>16</sup>. Therefore, the main challenge would be to establish some way of describing and modelling the harmony-seeking process, the fifteen properties and the idea of wholeness. A reduced number of authors 17-20 have attempted to embark on the path outlined by Alexander. Salingaros<sup>17</sup> have suggested the first mathematical treatment for the degree of life. His measures are quite simple and present some limitation that indeed was highlighted by Alexander<sup>14</sup> (pp.469-472). Ekinoglu and Kubat<sup>20</sup> and Jiang<sup>18, 19</sup> have proposed interesting methods for measuring wholeness, the former based on entropy measure and the latter based on a complex network approach.

In line with Jiang's research, this paper discusses the possibilities and drawbacks of operationalising Alexander's concepts from a network analysis perspective. We argue that there are much more possibilities to explore towards a configurational approach than previously exposed by Jiang<sup>18, 19</sup>. We emphasise that a network approach is just one of the possible ways to explore Alexander's ideas. Obviously, there is a diversity of possible approaches, although in a reduced number of scientific papers, as we have just referred above.

The aim of this paper is contributing to the discussion of how to operationalise the harmony-seeking and centring process based on network analysis. This kind of research is important because it can lead to the development of a tool for assessing wholeness, in other words, a tool for supporting urban design decisions. Firstly, we attempt to bring Alexander's theories closer to an urban design context. Moreover, we attempt to bring it closer to an urban network perspective since we are concerned about configurational issues of urban design. Then, we highlight the diversity of spatial network analysis available in the field of urban configurational studies. It is illustrated with a case study, where we apply network-modelling methods. Finally, we discuss possibilities of increasing the potentialities of a network approach to operationalise Alexander's concepts. The paper concludes with the main drawbacks, challenging issues and possibilities for future research.

#### Bringing Alexander's Concepts to an Urban Design Context

Alexander's theories, as previously exposed, are too abstract and generic to be straightforwardly applicable to urban design processes, or any kind of design process at all. Consequently, some interpretation is required, especially if we are committed to operationalising measures. In order to make his concepts more concrete, we assume some delimitation.

The first one is to discuss wholeness and the properties within an urban design realm. In fact, urban design is still a broad field to deal with, since it covers a wide range of scales and aspects. Seen in these terms, urban design can embrace small details from streetscape, buildings typology, green areas and activities distribution, networks of infrastructures and even entire cities—to cite just a few examples. Fortunately, the broadness of Alexander's work is not limited to a single scale. However, dealing with such a wide range of scales would be impossible. Thus, the scale addressed in the present study is the whole city scale, in other words, the urban planning scale. Defining a scale make it easier to define what are the subcomponents of the system, namely, the centres. If we are considering a city as the whole system, we can define the streets and built forms as the primary elements. Streets and buildings are precisely the key features in the structuring of urban morphology at this scale.

Several studies have shown that the configuration of the streets is a primary aspect in the structuring and dynamics of the cities, and it has been explored extensively through a network approach in the urban configurational studies<sup>21-25</sup>. Thus, the interconnectedness of the streets can be considered one of the most important aspects of the urban design in the scale we are addressing here. Similarly, the built forms can be considered as connected elements. Together with streets, buildings also contribute to give rise and to support the global character of the urban structure. Put it in other words, the built forms are as important as the street network, although studies attempting to include built forms are scarcer in the literature in the urban configurational literature<sup>26-28</sup>.

The main aspect we focus on is the configuration of the core elements of the urban structure: street and built forms. We are concerned about how the spatial elements connect to each other, as typically approached in configurational studies. Thus, we assume an urban network approach, in which the streets and built elements play a major role. Graphs are the principal mathematical language to describe properties of connected

components<sup>29, 30</sup>. Moreover, graph theory has been broadly used in urban studies<sup>6, 21, 23-28, 31-35, 39-40</sup>. This kind of work can be considered a particular field of urban morphology as it essentially addresses the configuration and the relationship between urban components. The main advantage is that the network properties of urban components can be mathematically manageable. In this sense, we argue here that this well-established research field can be useful to operationalise some of Alexander's abstract concepts. Besides, Alexander's recent theories are very suggestive of a network approach. According to Alexander, the degree of life of a given centre is defined not by the centre itself but by its position in the entire field of centres<sup>14</sup> (p.459). The author gives us others clues that make us believe that a network approach seems to capture the fundamental idea of wholeness and degree of life. Alexander suggests that each centre—as a bit of geometry in the space—affects and changes the other centres<sup>14</sup> (p.415).

Another bridge between Alexander's theories and a network approach is the scale-free property<sup>19</sup>. Some complex networks present far more less-connected elements than well-connected ones so that their degree of connectivity reveal a power law distribution. It is called a scale-free network<sup>36</sup>. Such property also has already been explored in urban studies<sup>25</sup>, <sup>32</sup>, <sup>33</sup>, <sup>35</sup>, <sup>37</sup>. The underlying structure and scaling hierarchy of artefacts are some of the key points in Alexander's theory, as previously exposed. In this sense, the network approach used in urban systems studies may be helpful to achieve it.

Finally, a network perspective on Alexander's wholeness has already been suggested<sup>18,19</sup>. Jiang defines wholeness as a hierarchical graph, in which centres are described as nodes and their relationships as links. The author suggests a mathematical model for wholeness, in which: a) PageRank score can measure the degree of life for each centre, and b) ht-index<sup>38</sup> can characterise the degree of wholeness for the whole system. PageRank (PR) score is an algorithm used to measure the centrality of websites. Jiang<sup>18</sup> applies it to measure the centrality of nodes in order to capture the hierarchy of the components. Ht-index is based on head/tails breaks, which is a clustering algorithm for data with heavy-tail distribution. It can be used for classification and visualisation of data, helping to characterise the levels of scale. Values are ranked in decreasing order and broken down by the average. Those above the average constitute the head and those below is the tail. This breaking process continues recursively for the head until the notion of far more small things is no longer present. Thus, ht-index can be defined as "the number of times that the scaling pattern of far more small things than large ones recur"38 and can be used for comparing different systems.

The main problem in using PR is that it grasps only one characteristic of urban structure and it is not clear whether it is the best way to capture hierarchy. In fact, it is not clear if there is a better way to capture hierarchy since there are several different methods to measure urban centrality and obtain spatial differentiation. Besides, each centrality measure reveals different distribution patterns<sup>33-35</sup>. Therefore, many measures other than PR could be used to identify hierarchy in a graph.

In this paper, we assume that Jiang's suggestion is true and that network analysis can bring a suitable mathematical language for dealing with Alexander's insights. In the following sections, we discuss possibilities of improving such approach, considering the diversity of techniques in urban configurational studies unexplored to the task of measuring wholeness.

#### **Urban Configurational Studies**

Typically, urban configurational studies use discrete components to represent the urban system, so that each component correspond to nodes and their connection links in a graph. There are three types of criteria used to define the discrete components<sup>39</sup>: a) preservation of geographic features, b) maximum morphological units, and c) minimum morphological units. In Figure 01.1, we can see one example of each criterion: a) intersections, which preserves geographic features—distances between nodes; b) axial lines, as maximum morphological units; c) street segments, as minimum morphological units of streets network.

The first one "a" is useful when Euclidean distances are important, because the corresponding graph is equal to the map, as we can see in Figure 01.1. This kind of representation is also known as a primal approach<sup>33, 34</sup>. Both "b" and "c" are known as a dual approach<sup>35</sup> since the graph is different from the map. In this case, Euclidean distances are deformed.

Descriptive systems like "b" and "c" give priority to streets network. The main difference between them is the level of aggregation of components. Maximum morphological units, such as axial lines<sup>21, 31</sup> and continuity lines<sup>40</sup>, are highly aggregated models because the primary units—the street segments—are merged according to a criterion, which is usually cognitive. Meanwhile, minimum morphological units can be viewed as a disaggregated form of axial representation<sup>39</sup>. Choosing the best descriptive system to model any problem is paramount because it strongly affects the results<sup>33-35</sup>, <sup>39</sup>. In fact, one of the crucial points to operationalise Alexander' is how to represent the parts of a system. The parts, namely the centres, are the