The Geometry of the Moiré Effect in One, Two, and Three Dimensions

The Geometry of the Moiré Effect in One, Two, and Three Dimensions

By

Vladimir Saveljev

Cambridge Scholars Publishing

The Geometry of the Moiré Effect in One, Two, and Three Dimensions

By Vladimir Saveljev

This book first published 2022

Cambridge Scholars Publishing

Lady Stephenson Library, Newcastle upon Tyne, NE6 2PA, UK

British Library Cataloguing in Publication Data
A catalogue record for this book is available from the British Library

ISBN (10): 1-5275-8405-4
ISBN (13): 978-1-5275-8405-1

To my children

"The moiré is everything"
an unknown Russian writer,
(unpublished)

TABLE OF CONTENTS

ACKNOWLEDGMENTS

I would like to appreciate my family, especially my elder daughter for valued linguistic guidance and my wife for practical comments. Their permanent help is always beneficial.

PREFACE

What is the moiré effect?

Have you ever walked along a fence line and suddenly noticed some strange pattern? Why do the wheels of a moving car or the propeller of a flying helicopter appear to rotate backwards or not moving at all, although you know the vehicle is moving forward? Have you wondered why the texture of digital photos or illustrations in a newspaper is seriously distorted sometimes? It is because of moiré patterns.

This optical phenomenon presents itself as repeated long-period darker and lighter bands of various shapes (wavy, rippled, circular, etc.).

- How are the moiré patterns formed?
- What causes them?
- When can the moiré effect be useful and beneficial rather than destructive?
- Can you recognize the moiré effect in 1 dimension?
- How would the moiré patterns appear in 2 or 3 dimensions?

The answers to above and other related questions are in the book. You will learn about characteristics of the moiré patterns in the spatial and spectral domains.

The book offers an opportunity to control the moiré patterns at a minimum (to improve the image quality) and at a maximum (to display images using the moiré effect). Also, we formalise necessary conditions for the moiré patterns to appear. Besides, we suggest a new definition of the moiré effect. In addition, we compare the moiré effect with other physical effects and devices which may look partially similar to it.

The book is based on author's research papers since early 2000s. We use the central projection, the model of the projection camera, and the rule of the incidence angles. Experimental photographs and computer simulation go with analytical calculations. Besides, the physical meaning of results is not overlooked.

The author informs on the geometric description of moiré patterns in several dimensions. Based on geometry of objects and their Fourier transforms (spectra), we address the key characteristics of moiré patterns in one, two, and three dimensions.

The book illustrates extra features of the moiré effect, which are usually not covered by other literature sources:

- the projected period as the basis for all dimensions,
- the rule of incidence angles,
- the spectral trajectories in 2D and 3D,
- the rotational Glass patterns in regular grids,
- the unchanged visible period in parallel orthogonal planes,
- the distance of the constant projected period in a cylinder,
- the static moiré patterns in the moving grids,
- the condition to observe the moiré patterns in chiral cylinders,
- the amplitude of the moiré effect,
- the probability of the moiré effect.

Equally important, the author found interesting characteristic features: the rational angles, the orientation of patterns at the moiré angle, the moiré mirror, and the constant visible period of the moiré patterns in parallel orthogonal planes. Also, in terms of the moiré period, the book describes the equivalence of the twist angle and the scale in non-identical parallel gratings and the quasi-periodic moiré patterns in periodic gratings.

The book aims to provide general guidelines for recognizing, understanding, and controlling the moiré effect. The author hopes it will also help researchers and engineers who may come across the moiré patterns in their practice and need to eliminate or refine them for the use in practical applications. In brief, there are about 150 figures, 250 equations, and 200 references in the book.

Specifically, the chapters of the book cover the signals and 1D arrays (stroboscopic effect, modulation of radio signals), the 2D moiré effect in coplanar images, and the 3D effect in generalized cylindrical surfaces. We study the effect in common 3D bodies. Examples of such bodies are a parallelepiped (box, two parallel surfaces of prism or another polyhedron), a prism (wedge), a sphere, and a cylinder. The time-dependent moiré patterns are considered including the spectral trajectories and the static patterns in moving grids.

The author intends the book for specialists (researchers and engineers) and public with prior background knowledge in mathematics and physics (undergraduate); an ordinary reader may find many interesting and sometimes unusual pictures of the moiré patterns.

INTRODUCTION

The moiré patterns in superposed (overlapped) arrays comprising repetitive lines, circles, or dots may appear as lighter and darker interference bands. One array is observed through another. On the one hand, black (opaque) lines of one array may cover the white (transparent) gaps between the lines of another array forming darker regions; on the other hand, coinciding black lines of both objects maintain the lighter regions.

When analysing the moiré effect, very essential is to acknowledge the works of many distinguished authors whose valuable contribution should not be left unnoticed: I. Amidror (2009), O. Bryngdal (1974), O. Kafri and H. Glatt (1990), K. Patorski and M. Kujawinska (1993), D. Post, B. Han, and P. Ifju (1994), C.A. Sciamarela (1982), and P.S. Theocaris (1969). Together with G. Oster, M. Wasserman, and C. Werling (1964), F.-P. Chiang (1979a), K. Creath and J.C. Wyant (1992) and other authors, whom I probably forgot to mention. Their significant ground-breaking works made a great impact on the theoretical and experimental investigation of the moiré effect. Of particular note is

> "a new pattern of alternating dark and bright areas which is clearly observed at the superposition, although it does not appear in any of the original structures", a quote from Amidror (2009).

The period of moiré patterns is longer than the periods of overlapping structures, as stated in the excellent Amidror's (2000, 2009) book. Particularly, the lines of both objects in Fig. 1 are nearly vertical, whereas the moiré patterns are almost horizontal. A careful reader can see that the moiré bands are formed on average.

In this book, we consider the geometry of the moiré patterns. We deal with the moiré effect in monochrome objects. Also, we examine planar and curved, one-, two-, and three-dimensional objects with arrays of repeated geometric elements on their surfaces.

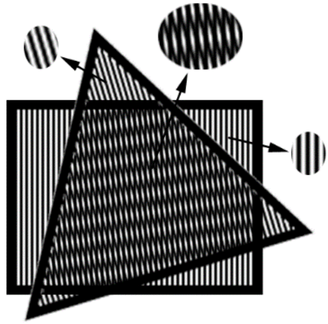


Fig. 1. Moiré patterns between twisted arrays (line gratings). © Springer, adapted from Saveljev (2016a) by permission.

One can find the moiré patterns virtually everywhere. Figures 2-4 show examples of the patterns in buildings and architectural details, in cylindrical and waved meshes, in overlapped grids, etc.

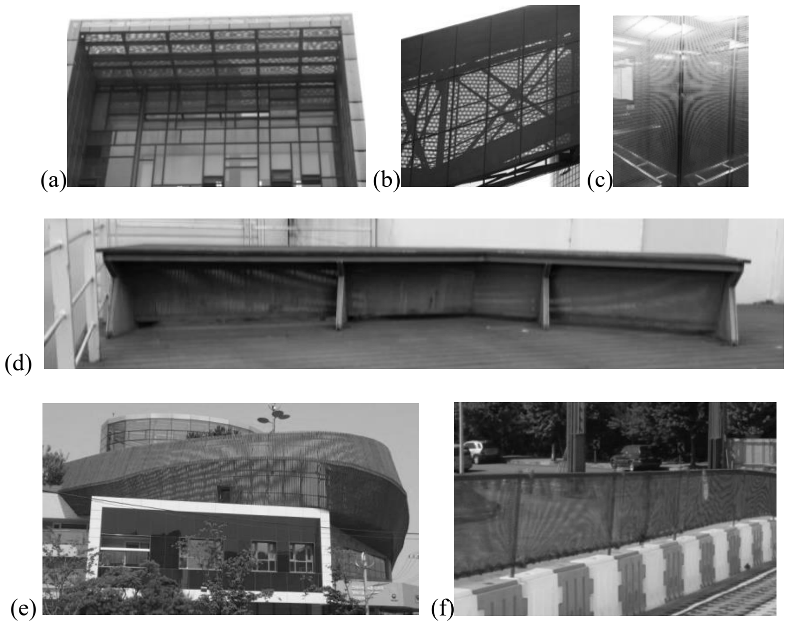


Fig. 2. Moiré patterns in buildings and constructions.

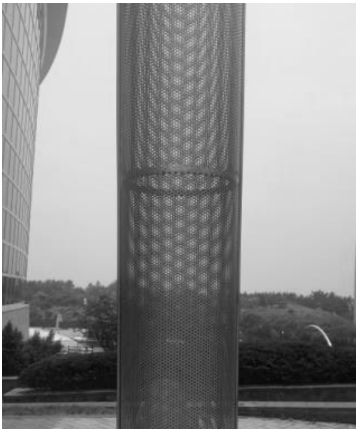
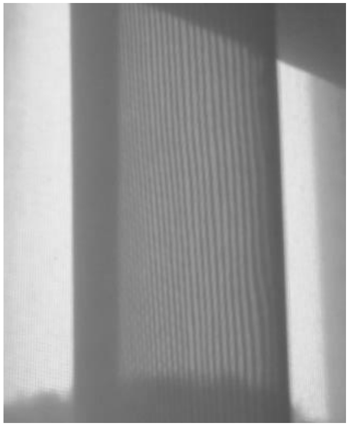
(a)  (b) 

Fig. 3. Moiré effect in meshed curved objects (cylinder and curtain).

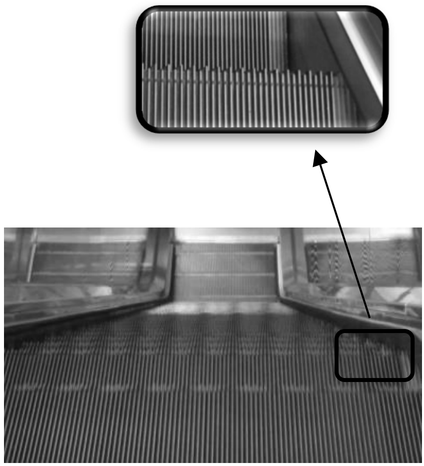
(a)  (b) 

Fig. 4. Moiré effect in partially overlapped edges of identical line arrays: (a) telephone booths and (b) steps of escalator.

The classic moiré effect is an optical interaction between superposed layers with periodically modulated transmittance (Chiang 1979a; Sciammarella 1982).

The moiré effect

> "occurs when repetitive structures (such as screens, grids or gratings) are superposed or viewed against each other." Quote from Amidror (2009).

We will refer to the above quotation by I. Amidror as the classic definition of the moiré effect in the images.

A grating modulates the intensity of the light. One can apply either the reflectance or transmittance function depending on a particular layout. Namely, for the gratings printed on paper, we use the reflectance function; for the gratings on a transparent film (such as OHP film), we use the transmittance function. The grating nearest to the observer must be transparent; other grating(s) may or may not be transparent.

The multiplication of the reflectance or transmittance functions describes the interaction between the overlapped gratings. This nonlinear operation produces new, longer periods.

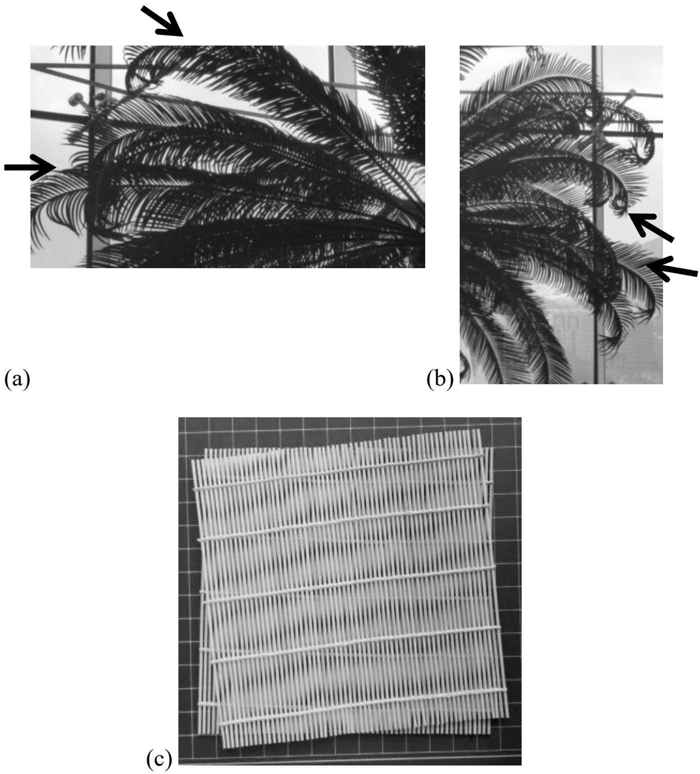
Analytically, one can describe the moiré effect using the convolution operation (Amidror 2009). The basic meaning of convolution is similarity. Correspondingly, the moiré effect is almost inevitable in regular or almost regular layers of similar structure. For example, the optical interaction between the elements of the halftone screen and the lines of the reproduced image may cause the undesirable patterns in reproduced halftone originals (Balasubramanian and Eschbach 2001). The moiré magnifiers (Hutley et al. 1994; Kamal, Volkel, and Alda 1998) also use the convolution principle.

Another definition from Kafri and Glatt (1990) is as follows,

> "The moiré effect denotes a fringe pattern formed by the superposition of two grid structures of similar period."

Note the word "similar" here and "repetitive" in the classic definition.

For the moiré effect, the periodical structure of the overlapped layers is important. Layers with periodic transmittance are gratings, grids, and meshes. We imply a periodic transmittance function, although the periodicity is not a necessary requirement (Glass 1969; Amidror 2007a). Therefore, the moiré effect in aperiodic or almost periodic layers is not impossible; see examples of the moiré patterns in bamboo mats and in plants in Fig. 5.



(a) (b) (c)

Fig. 5. Moiré effect in aperiodic structures: (a), (b) palm leaves, (c) bamboo mat. In (a), (b), some aperiodic moiré patterns are pointed by arrows.

In rough terms, the moiré effect shows the similarity of two similar arrays and magnifies the differences between them. The larger is the difference, the less is the similarity, and v.v. The similarity is maximal in two ideally aligned identical arrays, where the moiré patterns have the maximum period (theoretically, infinite). We can hardly observe such patterns. However, an angular misalignment can produce clearly visible moiré patterns. As the misalignment (the dissimilarity of positions) increases, the moiré bands become thinner and closer to each other. Their period becomes shorter, and finally, they happen to be unrecognizable.

The higher is the regularity of the layers, the stronger is the moiré effect, and the brighter are the moiré patterns. They often reproduce an enlarged structure of the layers, as stated by Hutley et al. (1994) and Amidror (2009), for example, the pixels of an LC display. The strongest effect appears with certain relationships between characteristics of layers satisfied. As we have seen, important is the geometric similarity of the layers. The strongest moiré effect appears with the maximum similarity. In particular, the research (Wu, Wu, and Chang 2013) confirms that the moiré effect in dissimilar layers (namely, in square and hexagonal grids) is practically unrecognizable.

The moiré effect is well investigated in optics (Oster, Wasserman, and Werling 1964; Sciammarella 1965; Bryngdahl 1976; Yokozeki, Kusaka, and Patorski 1976; Chiang 1979a; Kafri and Glatt 1990; Amidror 2009) including the optical measurements (Creath and Wyant 1992; Post, Han, and Ifju 1994; Li, Liu, et al. 2013). Note that the gratings involved in the moiré effect are not diffraction gratings, rather structures with a period of several millimetres or centimetres, which is much longer than the wavelength of the visible light. Therefore, one can describe the moiré effect in terms of linear optics, i.e., by rays.

In visual displays (dynamic screens, digital photographs, and printouts), the moiré effect may create an additional unwanted image over a useful image. The moiré patterns may appear in some unintended areas. Therefore, in displays, the moiré effect is an undesirable effect (Dohnal 1999; Bell Craig, and Simmiss 2007). This is especially important for autostereoscopic three-dimensional displays (Lee, Park, et al. 2016) because of their typical structure of periodic layers.

In contrast to displays, the metrology uses the moiré effect for highly accurate measurements (Bryngdahl 1974; Chiang 1975; Yokozeki and Patorski 1978; Chiang 1979a; Sciammarella 1982; Creath and Wyant 1992; Patorski and Kujawinska 1993; Post, Han, and Ifju 1994). Besides, one can find recent examples in a number of studies (Xie et al. 2004; Abolhassani 2010; Huang, Liu, and Xie 2013; Wen et al. 2017). Also, the moiré effect is efficiently used, for instance, in the document protection (Ostromoukhov et al. 1996; Hersch and Chosson 2004; Amidror, Chosson, and Hersch 2007b), the image encryption (Muñoz-Rodríguez and Rodríguez-Vera 2004; Ragulskis, Aleksa, and Saunoriene 2007), the three-dimensional displays (Nagasaki and Bao 2008; Wang 2010), the moiré interferometry (Yokozeki and Mihara 1979; Kujawinska, Salbut, and

Patorski 1991), not to mention the alignment (Zhou et al. 2008; Chen, Yan, et al. 2010), as well as in many other practical applications.

This is a moiré effect in the macro-world.

The rays of other nature may also produce the moiré patterns. If the multiplicative model of the interaction is appropriate (the multiplication of transmittance functions works well), other types of rays can also create the moiré patterns on different scales. Beyond optics, one should reconsider the transmittance function for those rays. For instance, the moiré phenomenon appears not only under the visible light but also in the X-rays (Riebel 1972; Bonse, Grae, and Materlik 1976; Fodchuk and Raransky 2003; Yoshimura 2015), in the electron beams (Bassett, Menter, and Pashley 1958; Glauert 1966; Su and Zhu 2010; Kim, Kim, and Lee 2015), and in the infrared light (Yao et al. 2007; Chen, Liu, et al 2015).

In the nanoworld, the moiré patterns have been observed in planar nanolayers under the electron microscope (Bassett Menter, and Pashley 1958; Pan et al. 2009; Yang et al. 2013) with the moiré period of several nanometres (Benedict et al. 1998; N'Diaye et al. 2006; Miller et al. 2010). Such nanopatterns are also known as moiré superstructures or superlattices (Balog et. al. 2010; Batzill 2012). The moiré effect in cylindrical nanoparticles (carbon and inorganic nanotubes and nanoparticles, single- and double-walled nanotubes) is not unknown (Suenaga et. al. 2007; Sadan et. al. 2008; Fukui et. al. 2009; Warner et. al. 2011).

In any layout, the particular appearance of the moiré patterns depends on the geometry of the layers (period, phase, and twist angle). In simple cases, such as coplanar periodic layers of simple structures, one can estimate the moiré patterns using the moiré magnifier concept stated by Hutley et al. (1994). The moiré patterns in parallel non-coplanar layers of similar structure reproduce the magnified and twisted structure of the layers (Saveljev and Kim 2012a). One can find examples in Figs. 1 and 6.

In order to observe the patterns, it is unnecessary to see the lines of the gratings themselves. The structure of the grating might be beyond resolution; and even if their too short period may be out of range (as in Figs. 2 and 3), we can see the patterns, anyway.

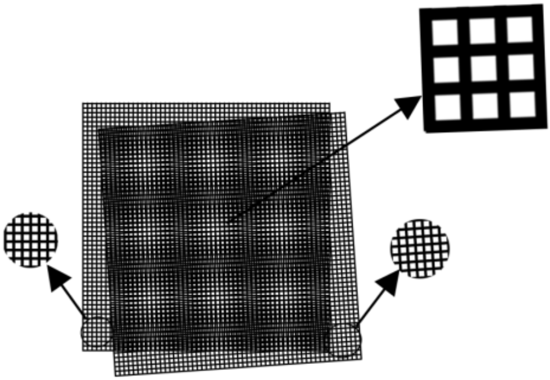


Fig. 6. Structure of moiré patterns is similar to the structure of the layers. © Springer, adapted from Saveljev (2016a) by permission.

Theoretically, the moiré effect always appears in the superposed similar gratings. Another matter is the visibility of the patterns, which might be high or low under different conditions. Particularly, it may depend on relationships between geometric characteristics of the repetitive structures, such as the orientation and the periods. As a result, we can characterize the moiré effect by the following features:

- First to mention is the point-by-point interaction in corresponding points, one point per layer. The resulting optical transmission is the product of transmittance functions of layers on the same ray.
- Typically, we observe the image of moiré patterns on a planar screen. (In visual observations, we may consider a small piece of the retina in the human eye as a flat surface.)
- There are at least two similar periodic structures overlapped, or placed closely (if non-periodic, anyway similar).
- The patterns are observed in rays passing through layers. Practically, it means an overlap of projections in two- and three-dimensional cases, and the direct overlap in one dimension.
- A new longer period that does not exist in either structure.
- The last but not the least is the averaging across several periods of the structures.

In the following, the wavenumber k is the reciprocal period λ of a periodic phenomenon (usually with the coefficient 2π),

$$k = \frac{2\pi}{\lambda} \tag{1}$$

Another characteristic is the wavevector. The modulus of the wavevector is equal to the wavenumber, but the vector has a direction.

It is well known (Amidror 2009; see also other cited articles) that the wavevector of the moiré patterns $\mathbf{k_m}$ is the sum or the difference of the wavevectors of the gratings $\mathbf{k}_1$ and $\mathbf{k}_2$, whichever of two mentioned operations results in the shorter wavevector,

$$\mathbf{k}_m = \min_{|k|}\left(\mathbf{k}_1 \pm \mathbf{k}_2\right) \tag{2}$$

In Eq. (2), the operator of minimum means the minimal modulus (the minimum length or the absolute value of the vector),

$$\mathbf{k}_m = \begin{cases} \mathbf{k}_1 - \mathbf{k}_2, \text{if} \left|\mathbf{k}_1 - \mathbf{k}_2\right| < \left|\mathbf{k}_1 + \mathbf{k}_2\right| \\ \mathbf{k}_1 + \mathbf{k}_2 \text{ otherwise} \end{cases} \tag{3}$$

The physical meaning of this formula is that the moiré effect generates the long-period (low frequency) component that is not present in either object (grating).

In the spectral domain, the wavevectors of the moiré patterns are near the origin. The zeroth approximation of that is the visibility circle defined by Amidror (2000). Typically, we assume that the radius of the visibility circle is shorter than the distance from the origin to the closest spectral component of either grating. In such a case, the gratings themselves (higher spatial frequencies) are unrecognizable, whereas the moiré patterns (lower spatial frequencies) can be visible.

Necessary conditions

Based on the above considerations, one may conclude that the moiré effect is a wide-area (averaged across several periods of gratings) point-by-point interference effect caused by an interaction (e.g., the overlap is the multiplication of the transmittance functions) in corresponding points of periodic (or just similar) layers; signals or volume structures are not prohibited. Also, a typical companion of such interaction is a low-pass filter (for example, an integrator). The summation is a linear operation, and therefore it does not mean an interaction, although sometimes, the result of summation might appear similarly. We intentionally use averaging (a sort of filter) to measure the period of the moiré patterns with higher accuracy (Saveljev, Son, et al. 2008). Besides, the research paper (Arpa, Süsstrunk, and Hersch 2017) shows how to use it effectively in retrieving information.

To summarize, two necessary conditions for the moiré patterns to appear are:

1) the interaction of similar structures and
2) the averaging.

The above conditions, together with the aforementioned properties, can produce a generalized definition as follows.

Definition of the moiré effect

The moiré effect is the effect of the formation of measurable patterns of a longer period caused by a point-by-point interaction in corresponding points between similar periodic structures of shorter periods and the averaging in the neighbourhood of those points.

To move forward, we have to understand the dimensionality of a physical effect. The dimension of the moiré effect is explicitly mentioned by Morimoto, Seguchi, and Higashi (1989), Khan (2001, chap. 6, p. 178), Chen et al. (2017), and Walger et al. (2019). If physical parameters vary along a straight line only (the coordinate on a ruler or the moment of time on the timeline), then it is a one-dimensional (1D) effect. The length or duration is the only coordinate of such effect. Then, if a single coordinate is not enough and we need two coordinates to describe the phenomenon (within a plane or an area on a map), it is a two-dimensional (2D) effect. The area needs two coordinates, for example, the length and the width. However, sometimes, we need the third coordinate, for example, the depth, in the description of a volume. Thus, all three spatial coordinates are involved, and it is a three-dimensional (3D) effect. For example, they typically use three coordinates (the length, the width, and the height) to describe an apartment building.

Predominantly, moiré effect is investigated in 2D (Sciammarella 1965; Yokozeki, Kusaka, and Patorski 1976). There are a few examples of the moiré effect research in 3D, e.g., a plane plus a curved surface (Creath and Wyant 1992; Li, Liu, et al. 2013). However, virtually nothing is available in literature on the 1D moiré. This book covers all three cases.

We will refer to a 1D planar structure comprising repeated parallel lines as a grating. A problem with several identically oriented coplanar line gratings is a 1D problem, because there is no change in the other direction, despite the number of gratings. Also, the time-dependent signals are definitely one-dimensional.

Then, we will refer to 2D structures (planar combinations of gratings) comprising repeated polygons as grids. Some of them are combinations of gratings, some are equivalent to the regular tessellations (Coxeter 1948; Grünbaum and Shephard 1990). The moiré effect in coplanar grids is the 2D problem. Sometimes, a grid is a combination of the line gratings (a square grid comprising two line gratings). In such case, one can probably reduce a 2D problem into a set of 1D problems.

Volumetric, non-planar structures or planar structures at different distances represent 3D problems. The moiré effect in 3D (for example, the moiré effect between a planar grating and a curved surface) is often used in measurements (Bryngdahl 1974; Chiang 1979a; Sciammarella 1982; Creath and Wyant 1992; Patorski and Kujawinska 1993). It means that non-flat or flat structures separated by a gap represent 3D problems (Sciamarella 1968; Chiang and Ranganayakama 1970). Non-planar meshed objects (like gridshells) also show the moiré effect in response to a periodic modulation of the transmittance across their surfaces.

Various essentially 3D cases of the moiré effect are investigated (Theocaris 1967; Garcia-Sucerquia, Carrasquilla, and Hincapie 2004; Fabbro et al. 2003; Travis et al. 2013; Saveljev 2018a). Particular examples of such surfaces are transparent surfaces of single-layered common 3D shapes, such as parallelepiped (two parallel planes), circular cylinder, sphere, and triangular prism (wedge consisting of two inclined planes). For instance, a transparent hollow cylinder is a model of a cylindrical single-layered nanoparticle (nanotube), which is essentially a non-planar 3D object (Derycke et al. 2002; Liu et al. 2005; Gao et al. 2006). The coaxial cylinders model demonstrates the moiré effect in the multilayered nanoparticles (nanophysics) and in the curved/flexible displays (optical engineering). Similar to the 2D case, one can sometimes reduce a 3D problem into a set of 2D problems.

With a few exceptions, the moiré effect is typically explored in the static setting. Some authors consider a dynamic or multiple-exposure effect (Liang et al.1979; Creath and Wyant 1992; Ragulskis, Maskeliūnas, et al. 2005). The study (Maskeliūnas et al. 2015) establishes the basic framework. The double-exposure interferometry with the optical filtration is presented in Chiang and Lin (1979b). Also, the double-exposure technique is further developed in Pokorski and Patorski (2010), especially using the stroboscope (Abramson 1996). Besides, the time-averaged moiré method is described in the application to vibrations (Ragulskis and Navickas 2009a) and visual security (Ragulskis and Aleksa 2009b).

One may consider the time as a fourth coordinate (in common with the relativity), and then probably refer the time-dependent 3D moiré patterns to as four-dimensional (4D) moiré. One chapter of the book considers the time-dependent effect, although in a reduced form (two spatial dimensions are involved). Regardless, one can naturally generalize it to 4D.

Let us briefly discuss the 2D and 3D cases. As previously mentioned, the moiré effect can be effectively used in measurements. However, in 2D visual displays, the moiré effect may create an unwanted image superposed with a useful image. The high sensitivity to the changed parameters makes the moiré phenomenon an adverse negative effect. Therefore, the moiré effect in displays is not acceptable. The fundamentals of the minimization stated by Amidror (2000) were further developed in Chen, Yeh, and Shieh (2008), Li, Zhang, et al. (2014b), as well as in other research articles. According to (Saveljev and Kim 2015a), to minimize the moiré effect by angle, one may use an irrational angle or select a rational angle with a low probability of the moiré effect. At the irrational angles, they did not observe the moiré patterns experimentally. Therefore, in such case, the probability is zero.

The minimization of the moiré effect in displays is one of the essential requirements for improvement of the visual quality, at least since the 1950s, in relation to the kinescopes (Ramberg 1952). Also, it is a major concern in printing (Amidror, Hersch, and Ostromoukhov 1994), image scanning (Davies, Harburn, and Williams 1986), X-ray imaging (Yamaguchi et al. 1989), LED displays (Lin et al. 2015), LCD projection displays (Jain et al. 2001), LCD back-light units (Olczak et al. 2006), and touch screens (Xu et al. 2017), not to mention autostereoscopic displays (Koike, Oikawa, and Utsugi 2006; Oku, Tomizuka, and Tanaka 2007; Kim, Park, et al. 2009). The elimination or at least reduction of the moiré effect is an important aspect in the improvement of the visual quality of 3D displays, especially autostereoscopic multiview and integral displays (Martinez-Cuenca et al. 2009; Hong et al. 2011; Kong, Jin, and Wang 2013). Such displays are typically designed with two layers of periodic structure (1D or 2D arrays of repeated elements) in the same way as Holliman (2006), Stern and Javidi (2006) having an integer ratio of the periods.

To prevent this undesirable effect, various methods are proposed, including optical filters (Okui et al. 2005), inclined lens arrays (Jang and Javidi 2002), the optimal angle in sinusoidal gratings (Saveljev, Son, et al. 2008) and in colour displays (Kim, Park, et al. 2009), as well as the inclined layout of pixels (Takaki, Yokoyama, and Hamagishi 2009). The main approaches to minimize the moiré effect are the direct analytical minimization (Creath and Wyant 1992; Saveljev, Son, et al. 2008) and the spectral minimization (Zhou et al. 2008; Amidror 2009). Also, the dual sampling (Yamaguchi et al. 1989) is applied; along with the Fourier transform (Amidror, Hersch, and Ostromoukhov 1994; Sidorov and Kokaram 2002) and the wavelet transform (Pokorski and Patorski 2010). Sometimes, the colour of the moiré patterns is considered (Amidror 2009; Kim, Park, et al. 2009). Despite that, this topic requires additional investigation.

For convenience, we define the moiré magnification factor μ as the ratio of the moiré period λ_m to the period of one grating λ_1 in the same way as Saveljev, Kim, and Kim (2018b),

$$\lambda_m = \mu\lambda_1 \tag{4}$$

One can apply the moiré magnification factor (the coefficient of proportionality) to the periods of the moiré patterns and the gratings, if they are proportional. This concept was inspired by the research papers (Hutley et al. 1994; Kamal, Volkel, and Alda 1998). In this book, we will regularly use the moiré magnification factor instead of the moiré period.

We arranged the book as follows. The necessary conditions and the generalized definition are given in the current section. The Glossary explains the usage of terms in this book.

Chapter 1 describes the 1D moiré effect in the optical gratings and the electrical signals. We provide formulas and examples, including the modulated signals. One of the most fundamental characteristics of any physical effect is its amplitude. We will discuss this highly important topic in Chapters 1 and 2, although it is rarely taken into consideration in relation to the moiré effect in literature.

Then, we describe the 2D moiré effect in the images in Chapter 2. It is the moiré effect in the coplanar grids. Some grids are superpositions of gratings. The amplitude, visibility, and spectra are considered.

The 3D moiré effect is the principal topic of Chapter 3. We explain the rule of the incidence angles for generalized cylindrical surfaces and obtain the formulas for the projected period of a single facet and the formula for the moiré period in overlapping facets. Then, we apply these principles to common geometric shapes (prism, parallelepiped, and cylinder). We also describe qualitatively the moiré effect in a sphere and in a volumetric 3D array.

Chapter 4 illustrates the optical time-dependent moiré effect. It is best demonstrated in the context of 2D objects and the time, implying a further extension to three spatial dimensions. Particularly, the spectral trajectories are presented together with the regular structures in the spectra, which can generate the quasiperiodic moiré patterns.

Chapter 5 describes the control over the moiré effect. Specifically, the minimization means an improvement of the image quality; the maximization means making a display. The moiré statistics is included. Examples of other physical effects similar to the moiré effect are also provided.

Finally, an interesting, practically useful issue related to the moiré effect in cylinders is described in Appendix.

List of Abbreviations

ADC	analogue to digital converter
AM/FM	amplitude/frequency modulation
BW	black and white; typically about colourless images/displays, but not about printer inks
CW/CCW	clockwise/counter clockwise; directions of rotation
DC	literally "direct current"; practically used to point to a static component, a constant bias
FWHM	full width of a curve at the height of half-maximum
GCS	generalized cylindrical surface; generalized means not only a circle, but any other line can define the base of such cylinder
LC	liquid crystal; typically it is about displays, for instance, a LC display (LCD)
LED	light-emitted diode often used in modern displays
LOS	line of sight
MTF	modulation transfer function
OHP	overhead projector which shows slides
PWM	pulse-width modulation
RC circuit	circuit consisting of a resistor (R) and capacitor (C)
SLM	spatial light modulator
SWNT/DWNT	single-/double-walled nanotube
TEM	transmission electron microscope

CHAPTER ONE

ONE-DIMENSIONAL MOIRÉ

1.1. Moiré Effect in Functions of One Variable

A function $f(x)$ of one variable x defines the output (the value of the function) for every x in the domain. Both domains of the input (the argument) and of the output (the value) are one-dimensional. The variable can be either a spatial coordinate (length) or time (duration). In this chapter, we consider the moiré effect in 1D objects such as linear arrays and the time-dependent signals.

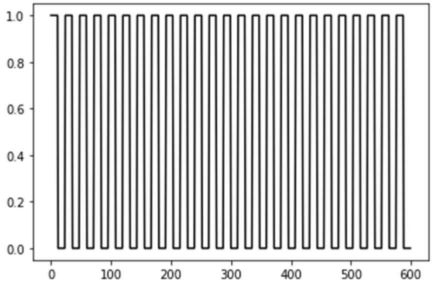
a) Linear arrays

It is not unknown that the amplitude A, the wavenumber k, and the phase φ characterize a planar sinusoidal grating of the infinite size (boundary conditions do not matter),

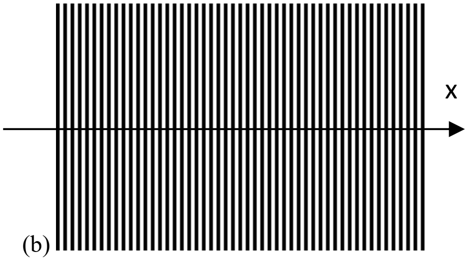
$$F = A\sin(kx + \varphi) \tag{1.1}$$

Then, the fundamental wavenumber together with higher harmonics additionally characterizes a grating of an arbitrary non-sinusoidal profile. In this chapter, we will often omit the word "fundamental" without losing essential details; we will say "wavenumber" with no regard to the profile, sinusoidal or non-sinusoidal. Note that the wavelength of the sinusoidal wave (or the period for a non-sinusoidal wave) is equal to the reciprocal wavenumber with a coefficient; see Eq. (1) in Introduction.

Consider a printed line grating with parallel lines of equal black (opaque) and white (transparent) areas arranged along a straight line (the abscissa in Fig. 1.1). This is a binary grating without intermediate levels.



(a)



(b)

Fig. 1.1. 1D objects: (a) 1D array, (b) line grating.

Consider two superimposed gratings. The first grating is printed on the paper; its underlying surface (the paper) is white. The second grating is printed on a transparent film and assumed to be thin. Correspondingly, we can consider two gratings practically in the same plane. Alternatively, one can use a transparent film for both gratings and place them on the white paper. In both cases, the visual result is the same.

In the case of an ideal alignment of the identical gratings (when the lines of both gratings are exactly parallel), the moiré patterns do not seem to appear. No recognizable patterns of a finite period can be observed. Instead, we can either see a uniform area in Fig. 1.2(a) or even nothing new at all in Fig. 1.2(b).

Therefore, in this chapter, we consider the gratings of different periods. Let’s find out what happens when such gratings are overlapped, see Fig. 1.3, where the second grating is a scaled copy of the first grating with a slightly longer period.