A Global Approximation Interpretation of Quantum Mechanics

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Foreword

At the end of 1986, I was a sophomore in college. My teacher talked a little about quantum mechanics in the general physics class. When I first saw the Schrödinger equation, I was mesmerized, staring at the equation for a long time. I have never come across a mathematical expression and don't know what it means. Later, I went to the library to find textbooks on quantum mechanics, but I was even more confused. I wanted to figure out everything, so I began to study physics courses by myself. I was an engineering student, and I spent almost all my spare time in the library. In addition to physics textbooks, I also read books on other subjects.

When it arrived time to prepare for the postgraduate entrance examination, I looked for the graduate program brochures of different universities in the library. I found that only Peking University had a graduate program in quantum mechanics. So I applied for the exam, and the advisor is Professor Zeng Jinyan. His textbook is one of the few ones I can comprehend when studying by myself. I didn't expect admittance from any university on my first try. Surprisingly, I was fortunate. Even though I prepared wrongly in one course, I did well in quantum mechanics and electrodynamics. I was admitted to Peking University after the interview. The year was 1989, and it was an eventful year. A lot happened in my family, China, and the world. The system didn't function properly. In July, I should have received the admission letter as the semester began in September. I didn't know I could continue with my study until October.

I could not concentrate on the problems that puzzled me during my postgraduate studies. Professor Zeng said that it is challenging to deliver progress in fundamental issues in quantum mechanics, and there must be progress to ensure graduation. So for a long time, I worked on nuclear structure theory. During my Ph.D., the concept of quantum information began to heat up. My doctoral dissertation was about the phase problem in quantum mechanics, but the focus of my work was still on the nuclear structure theory. When I was a postdoctoral fellow in the Institute of Theoretical Physics at the Academy of Sciences, I also worked on nuclear theory. Some of the students taught by Prof. Zeng at Tsinghua University have turned to quantum information. I participated in some discussions and read some literature myself, but I quickly became confused and felt something was wrong. An obvious question is, isn't quantum uncertainty intrinsic? Why is it not reflected in quantum information?

Later, I returned to work at Peking University, in charge of the computer lab, teaching, going abroad, entering the field of supercomputing and fusion, and slowly leaving quantum mechanics research. However, I never stopped thinking about quantum mechanics' puzzling issues and paid attention to related research and progress. I was skeptical of some conclusions claimed and began investigating those fundamental issues. Around 2010, I already had some opinions. In 2015, I felt that maybe it was time to talk, so I had a graduate student (Yiwen, the co-author of this book) and began to prepare for the new interpretation. I did realize that it would be an arduous journey to challenge the established Copenhagen doctrine. In 2017, we wrote the first paper on quantum entanglement (QE), arguing that QE is a common quantum coherence phenomenon and proposing an experimental verification method. Since people believe that Bell experiments have fully proven quantum non-locality, those who question it simply do not understand it themselves, and our paper cannot be published. Although their reasons against us are not tenable, convincingly explaining QE does involve further clarifying the definition of elementary particles, the physical image of photon, reality, coherence, etc.

By 2019, I was confident that the new interpretation should have covered most issues, so I named it the Global Approximate Interpretation (GAI) but had to prepare a long article. At the end of 2019, the Covid-19 epidemic began. At the beginning of 2020, the international and domestic political trend has undergone severe changes. I feel that China is heading in the wrong direction, and I should speak up. In my opinion, China is a part of the world and should get along with other countries equally and be friendly, and respect, appreciate and learn from other countries. In the days of the epidemic of isolation, I began to speak on the Internet. From the perspective of the development process of human civilization, I talked about the importance of democracy, tolerance, diversity, humanity, integrity, and totalitarian evil. My remarks rippled, and I was silenced many times and attacked by Internet thugs. The thugs reported me to the propaganda police, putting me under tremendous pressure. They attacked me on all fronts, saying that I was a crank against quantum theory, a traitor, and a disgrace to my university. In the middle of 2020, I began to prepare GAI in chapters and put it on the net. I did not get many positive responses from the academic community, and the pressure from other aspects increased. Even some of my friends suggested that I was just a person who opposed everything.

For a while, under pressure, I doubted myself because I had to respect the facts and the opinions of others. I must seriously consider whether I was insane or my theory is stupid. I made a long list which is the pieces of evidence of my insanity or stupidity. Of course, I also have the opposing argument, but how could I ensure neutrality in defending myself? From my understanding, the theoretical footings of quantum information are fragile, so I was unpopular in the quantum information community. However, most people believe in "authority" more. In their opinion, how could I, an associate professor, challenge so many professors and academicians? People politely kept their distance from me. I have to respect that distance. To avoid embarrassment, I must be more sensitive to their attitudes and even actively distance them. These actions made me timider and ridiculed when I took my stand.

But I still have confidence in GAI. After all, if I give up, doesn't that prove my criticizer's allegations against me are false? How can an insane crank give up easily? I believe that Einstein, Schrödinger, Weinberg, and even Bell would be happy to know our interpretation. We have listed some experiments supporting GAI, but the interpretation of QE needs to be tested by further experiments. Although it has been four years, no relevant experiments have happened.

At the beginning of 2021, I finished the paper, put it on the Internet, and sent it to some friends and colleagues, but there were very few responses. The positive responses came mainly from retired scientists. It has been replenished and revised for a year, and I think it should be clear enough.

First of all, I would like to thank Prof. Zeng Jinyan for bringing me into quantum mechanics research and for all his help and care, together with his wife, Mrs. Guo Jiyu, over the years. Thanks to Liu Yuxin, Gao Yuanning, Chen Xiaolin, Qin Shaojing, Li Xueqian, Li Shengtai, etc., for their discussions and support. Thanks to my family for their constant love and support. Finally, I would like to thank Ms. Helen Edwards of Cambridge Scholars Publishing for her help with this publication.

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June, 2022

List of Abbreviations

Abbreviations	Full words
CCD	charge coupled device
CFD	computational fluidal dynamics
CI	Copenhagen Interpretation
GAI	Global Approximation Interpretation
GUT	Grand Unification Theory
NRA	non-relativistic approximation
QE	quantum entanglement
QED	quantum electrodynamics
QM	quantum mechanics
SE	Schrödinger equation
SI	Standard Interpretation
SM	Standard Model
STR	special theory of relativity
TOE	Theory of Everything

Starting questions

What is quantum? Can I have one?

What are particles? In particular, what are photons?

What is wave-particle duality? Does it meet the requirements for a scientific definition?

Why is quantum mechanics difficult to understand? Was it complete when it was established in the beginning?

What is the Schrödinger equation? Why is there no interpretation for the equation?

What is quantization?

Are fundamental physical quantities continuous or quantized, such as space-time and energy?

Why do there exist two sets of theories, quantum and classical? Are they both right, or neither?

Is there causality in quantum entanglement? If there is not, can it still be called entanglement?

What is objective reality? Does it have only two answers, existence and nonexistence?

In a nutshell

Quantum mechanics is the most glorious and controversial theory of the physical world. The interpretation of QM has been fiercely debated ever since its early times. People are still puzzled about the understanding of many quantum behaviors. Instead of the quantum wave function and measurement, we found that the properties and physical meanings of the Schrödinger equation are the keys to understanding all quantum behaviors, i.e., the globality and the approximation of the equation, as well as the method of equation solving. With the additional knowledge of fundamental interactions and elementary particles, we can intuitively understand all quantum properties by putting back the omitted parts in the approximation. Global Approximation Interpretation (GAI) explains well the quantum wave function, the origin of the probability interpretation, coherence, measurement, the boundary between the classical and quantum world, the emergence of quantization, the properties and the physical pictures of elementary particles, quantum entanglement, quantum eraser experiment, etc. GAI defines measurement based on interactions and finds that reality is relative. Formerly contradictory philosophical theories can reconcile, such as reductionism and holism, idealism and materialism, and determinism and indeterminism. The philosophical difficulties posed by the Copenhagen Interpretation are all gone.

Chapter 1

Introduction

1.1 A brief history of quantum mechanics

In 1900, to understand blackbody radiation, Planck suggested that energy can only appear in quantized amounts (Planck and Dtsch., 1900). In 1913, to explain the Rydberg formula of atomic spectral lines, Bohr suggested that electrons can only rest on the "quantized" orbits of atoms and only absorb or release "quantized" energy (Bohr, 1913). In 1923, de Broglie proposed the concept of matter-wave (Broglie, 1923). In 1926, Schrödinger wrote down the Schrödinger equation from the concept of matter waves (Schrödinger, 1926a), and explained the spectral lines and covalent bonds of hydrogen atoms. Finally, in 1927, Bohr and Heisenberg formulated the Copenhagen Interpretation (CI) of quantum mechanics (Born, 1926; Heisenberg, 1927; Jordan and Klein, 1927) to explain the wave function. Since then, quantum mechanics in the microscopic world has been fully established, and the interpretation of the Copenhagen School is still the orthodox doctrine today (Wimmel, 1992).

The main principles of CI are: wave-particle duality, with all quantum properties in its wave function; the quantum wave function is probabilistic, and measurement causes it to collapse; wave-particle duality satisfies the principle of complementarity; the connection to classical physics is the principle of correspondence (Bohr, 1920); some related physical properties obey the Heisenberg uncertainty principle.

The rise of the CI of quantum mechanics shocked the scientific community and was very successful, but it also met fierce criticism. For example, Einstein, Schrödinger (Schrödinger, 1935), and others are firm opponents of some of these concepts, such as probability and quantum wave function collapse.

Although many other interpretations have appeared in the past hundred years, such as the pilot wave theory (Broglie, 1927), the multiverse theory (Everett, Dewitt, and Graham, 1973), etc., CI has always been prevailing.

With the progress of Bell experiments since the 1980s, the recognition of CI reached the highest level (Aspect, Grangier, and Roger, 1981). In the following decades, although the recognition has declined (Norsen and Nelson, 2013), it remains the basic interpretation taught in textbooks. Theoretical physicists, such as Feynman (Feynman, 1967), Smolin (Amelino-Camelia, Freidel, Kowalski-Glikman, and Smolin, 2011; Smolin, 2013), 't Hooft ('t Hooft, 2000, 2019, 2020a,b), Weinberg (Weinberg, 2017), etc., have all expressed their concerns on QM and CI.

QM does not answer questions such as "what are elementary particles?" and "what is the interaction?" These issues are the frontiers of subsequent research. The progress of these studies has dramatically changed our understanding of the world. Although we cannot say that these theories are final, we should put these understandings back into quantum mechanics to solve the problems such as "no one understands quantum mechanics" (Feynman, 1967).

The centerpiece of QM formulation is the Schrödinger equation (SE). However, no one explained the origin of the equation, and it is absent in CI.

We will re-examine the basic properties and the physical implications of SE and its solution (quantum wave function) based on the pictures of elementary particles and interactions and establish a quantum mechanics interpretation based on these fundamental theories.

1.2 Confusing concepts

Unlike almost all scientific theories, many concepts and phenomena in QM are quite confusing, and we list some below. Some of them have been heatedly debated (Bohr, 1931; Heisenberg and Maclachlan, 1958; Mehra and Rechenberg, 1982), and some first appear in this book:

1. **Wave-particle duality**

   Is "particle" a wave or a particle? What is meant by "sometimes behave like a wave and sometimes a particle"? Waves distribute coherently in a space-time continuum, and particles generally have definite sizes and positions but no coherence. Any object is *both* a particle and a wave, but *either* a particle or a wave. This duality is a vague definition, which should not appear in a scientific theory.

2. **Schrödinger equation**

   As the core of QM formulation, what does SE mean? Schrödinger himself never explained how he got it and what it means, but it worked perfectly.

3. **Measurement problem**

   Before and after the CI measurement, the process is discontinuous. The transition occurs instantaneously and randomly, which can cause all physical quantities to be discontinuous (Schlosshauer, 2003; Wigner, 1995), such as energy and angular momentum, but these quantities should be conserved. In the classical system, the measurement does not affect the system's state. However, in the quantum system, the measurement causes a sudden and irreversible change to the system (Bub and Pitowsky, 2008).

4. **Uncertainty principle**

   In classical physics, the properties that particles can have simultaneously, such as position and velocity, are impossible to obtain simultaneously in quantum mechanics. Thus, the more precise one quantity is, the more uncertain the other quantity must be (Heisenberg, 1927).

5. **The puzzling properties of the elementary particles**

   For a classic object, we can measure its various physical properties. These properties are self-consistent, with clear physical meaning, and understandable, such as size, momentum, mass, angular momentum, etc. However, in quantum mechanics, we have to artificially define the basic properties of elementary particles and regard them as intrinsic ones. They are hard to understand, and there is no physical picture. For example, what is the size of an electron? Its surface may rotate faster than the speed of light (Hey and Walters, 2003). And what is a photon?

6. **Reality**

   If you don't measure a quantum object, you cannot know its properties, but measurement changes it. So then, before the measurement, does it have an objective state (de Muynck, DeBaere, and Martens, 1994)? Einstein thought there was, Bohr thought not, or it is impossible to know. "If I don't look at the moon, will it not exist? (Mooij, 2010)" is the exaggeration of this argument.

7. **Quantum entanglement (nonlocality)**

   Two entangled particles can affect each other instantaneously, even if they are far apart. This image violates the principle of locality (Einstein, Podolsky, and Rosen, 1935), but supporters claim that countless experiments have irrefutably proved this phenomenon (d'Espagnat, 1979; Gröblacher, Paterek, and et al., 2007).

8. **Determinism**

   Classical physics is deterministic in principle. That is, the physical state of the past determines the future. In quantum mechanics, the future can only be a possibility. Quantum mechanics can only tell what could happen in the future. It gives the probabilities of each happening. Of course, there are some other interpretations, such as multi-universe interpretation (Everett, Dewitt, and Graham, 1973).

9. **Delayed-choice experiment**

   Quantum interference experiments are confusing enough. In the delayed-choice experiments (Kim, Yu, Kulik, Shih, and Scully, 2000; Wheeler, Zurek, and Ballentine, 1983), it seems that future choices can change the past paths of the particles. The experiments seem to support the idea of changing the past in the future.

10. **Photon problem**

    As a particular case where the properties of elementary particles are hard to understand (Risby, 1999), photons are especially incomprehensible and even theoretically inconsistent. For example, does a photon have specific energy? Each answer has its agreements and contradictions with experiments (Landau and Lifschitz, 1977).

1.3 Global Approximation Interpretation

CI of QM has been challenged ever since its beginning. It will be strange that a controversial theory is correct the first time people constructed it, and no improvement is possible.

Global Approximation Interpretation (GAI): take advantage of our later understanding of elementary particles and fundamental interactions established by quantum field theory (Standard Model (SM)); explain the globality and the approximation implied in Schrödinger equation; investigate the global nature of nonrelativistic QM; discusses measurement from the perspective of interaction; offers an extensive set of interpretation of quantum mechanic concepts.

We can also name it Standard Interpretation (SI) because its theoretical foundation is SM.

GAI (SI) can provide a physical picture consistent with cognitive intuition for all the above annoying concepts.

Chapter 2

Standard worldview

2.1 Elementary theories of the physical world

How does everything work? After thousands of years of hard work by countless thinkers and scientists, we should have almost figured out the fundamental laws of interactions between objects.

The current most recognized theory about elementary interactions is Standard Model (SM) (Cottingham and Greenwood, 1998). It describes the three fundamental interactions in the universe: electromagnetic, weak, and strong interaction, excluding gravity. We can integrate the three kinds of interactions, called the Grand Unification Theory (GUT) (Ross, 1984). If we put gravity in, then it is called the Theory of Everything (TOE) (Laughlin and Pines, 2000).

We will not go into the details of these theories, but numerous experiments and reasoning have rigorously tested them. We do not have the final theory yet, and the current one does not necessarily have to be right in every detail. The theories may be subject to change in the future. However, we can still draw some lines of these fundamental laws that govern the physical world. What should our worldview look like under the SM (GUT)?

Unlike our general concept of concise and graceful physical laws, real-world interactions are extraordinarily complicated and trivial. We can look at a simplified version of the Lagrangian of the SM of particle physics (Cottingham and Greenwood, 1998):

$$
\begin{aligned}
L = & -\frac{1}{4}B_{\mu\nu}B^{\mu\nu} - \frac{1}{8}tr(W_{\mu\nu}W^{\mu\nu}) - \frac{1}{2}tr(G_{\mu\nu}G^{\mu\nu}) \\
& + (\bar{\nu}_L, \bar{e}_L)\bar{\sigma}_i^{\mu} D_{\mu} \begin{pmatrix} \nu_L \\ e_L \end{pmatrix} + \bar{e}_R \sigma^{\mu}{}_i D_{\mu} e_R + \bar{\nu}_R \sigma_i^{\mu} D_{\mu} \nu_R + (h.c.) \\
& - \frac{\sqrt{2}}{\nu}[(\bar{\nu}_L, \bar{e}_L)\phi M^e e_R + \bar{e}_R \bar{M}^e \bar{\phi} \begin{pmatrix} \nu_L \\ e_L \end{pmatrix}] \\
& - \frac{\sqrt{2}}{\nu}[(-\bar{e}_L, \bar{\nu}_L)\phi^* M^{\nu} \nu_R + \bar{\nu}_R \bar{M}^{\nu} \phi^T \begin{pmatrix} -e_L \\ \nu_L \end{pmatrix}] \\
& + (\bar{u}_L, \bar{d}_L)\bar{\sigma}_i^{\mu} D_{\mu} \begin{pmatrix} u_L \\ d_L \end{pmatrix} + \bar{u}_R \sigma^{\mu}{}_i D_{\mu} u_R + \bar{d}_R \sigma^{\mu}{}_i D_{\mu} d_R + (h.c.) \\
& - \frac{\sqrt{2}}{\nu}[(\bar{u}_L, \bar{d}_L)\phi M^d d_R + \bar{d}_R \bar{M}^d \bar{\phi} \begin{pmatrix} u_L \\ d_L \end{pmatrix}] \\
& - \frac{\sqrt{2}}{\nu}[(-\bar{d}_L, \bar{u}_L)\phi^* M^u u_R + \bar{u}_R \bar{M}^u \phi^T \begin{pmatrix} -d_L \\ u_L \end{pmatrix}] \\
& + \overline{(D_{\mu}\phi)} D^{\mu}\phi - m_h^2[\bar{\phi}\phi - v^2/2]^2/2v^2
\end{aligned}
\tag{2.1}
$$

The terms in the righthand side of the above equation are (from the first line to the bottom, respectively): (a) U(1), SU(2), and SU(3) term; (b) lepton dynamical term; (c) electron, muon, and tauon mass term; (d) neutrino mass term; (e) quark dynamical term; (f) down, strange, and bottom quark mass term; (g) up, charm, and top quark mass term; (h) Higgs dynamical and mass term.

The Lagrangian in quantum field theory represents the dynamics and kinematics properties of the system. It describes the number of degrees of freedom in the system, the energy distribution, and transition rules in and between each degree of freedom. Each particle is a dynamic field distributed in space-time.

The space-time of quantum field theory is a covariant that complies with special relativity, so all the fundamental interactions of the SM have to obey the principle of locality (Einstein, 1948), that is, the propagation of any interaction cannot exceed the speed of light.

When calculating actual physical processes, Feynman's rules of path integral are applied (Feynman, 1948). In principle, even for the simplest single-particle case, there is an infinite number of orders of terms. Every next order is much more complicated than the previous one.

Lagrangian, or the terms in the field equations, contain various differential operators. The physical quantities must be continuous for the differential operator to be valid. All quantities in different degrees of freedom in the field equation must be continuous.

The Feynman rule (Feynman diagram) of quantum field theory calculations (Peskin and Schroeder, 1995) addresses the contribution of various possible particle interaction rules to a specific process. Generally, the calculations go well only when the perturbation assumption holds, or the higher-order contribution is negligible. In most cases, the perturbation assumption is invalid. Even if the perturbation assumption is valid, the high-order processes may be too complicated.

The formulation, implication, and calculation of the standard model of quantum field theory all show that our physical world is a very complex, intertwined, and nonlinear one (Boeing, 2016). Any elementary process is nonlinear, and even the vacuum itself is infinitely complex and nonlinear (Alvarez-Gaumé and Barbón, 2001).

As for the GUT, and the TOE, although we still don't know their specific form, they must include all interactions, and all matter must participate in all fundamental interactions, and they are all coupled together. So their formulation and implications might even be more complicated than the above.

We can give a simple example to illustrate the complexity of interactions between matter, even considering the classical electromagnetic interaction only. All matter participates in electromagnetic interactions. Even if no charge is involved, there is always a spin and magnetic moment. An electron has an infinite range of action, so any charged object will inevitably influence infinity. If it is neutralized, such as hydrogen atoms, will the affected space be tiny? No, its range of influence is still infinite because it has a magnetic moment. According to its internal energy configuration, there are higher-order electrical multipole momentums, and the effective range of action of these momentums is also infinite. If you consider the changes over time, the changes caused by changes, and the actual complexity of the physical world, any precise calculation or measurement is impossible.

2.2 Summation

The physical picture of the world based on the standard model can be called the standard worldview.

Whether starting from elementary physical theory or our common sense, we find that the physical world is highly complex, with multi-degree-of-freedom, nonlinear, and possesses infinite objects. As a result, any seemingly simple process is complicated.

Some observations:

- All fundamental interactions obey the principle of locality, and there is no superluminal interaction (Einstein, 1952).
- The mathematical formulation (Marlow, 1978) is not the actual physical world. The physical world is inherently complex, and mathematical formulation can only approximate a specific direction to a certain extent.
- The equivalence between matter and energy has profound significance. In the current theoretical framework, matter or energy is the basis for the physical world's existence or observability.
- The Lagrangian, or the laws of physics, are the same for any space-time point. Every point in space-time is equally complex. As long as the conditions are the same, such as energy density or other degrees of freedom, the physical processes are the same.
- The many differential notations in the SM Lagrangian mean those quantities have to be continuous.
- *Energy density* is the key to physical appearance. Different energy scales show different physical properties.
- Every particle has the contribution of the full-space space-time point, or every particle is a global mode.

Chapter 3

The physical meaning of Schrödinger equation

3.1 Importance and derivation of SE

SE is the fundamental centerpiece of QM formulation, but how did the equation come about? According to Feynman, "It is not possible to derive it from anything you know. It came out of the mind of Schrödinger." (Feynman, Leighton, and Sands, 1965). Schrödinger never explained how he got his equation. SE remains a fundamental assumption in QM. However, CI did not mention it at all. It focused on interpreting the derived solution of SE, i.e., quantum wave function.

However, there is a widely circulated "derivation of Schrödinger equation", which appears in some quantum mechanics textbooks (Zeng, 2013). In which matter-wave (quantum) is in the form of an ideal plane wave:

$$\psi(\mathbf{r},t) \sim e^{i(\mathbf{k}\cdot\mathbf{r}-\omega t)} = e^{i(\mathbf{p}\cdot\mathbf{r}-Et)/\hbar}, \tag{3.1}$$

by finding the operator associated with energy and momentum,

$$E \to i\hbar\frac{\partial}{\partial t}, \quad \mathbf{p} \to \hat{\mathbf{p}} = -i\hbar\nabla \tag{3.2}$$

and substituting them into classic energy expression,

$$E = \frac{1}{2}p^2 + V(\mathbf{r}), \tag{3.3}$$

we have SE:

$$i\hbar\frac{\partial}{\partial t}\psi(\mathbf{r},t) = \left[-\frac{\hbar^2}{2m}\nabla^2 + V(\mathbf{r})\right]\psi(\mathbf{r},t) \tag{3.4}$$

This derivation gives SE the correct form, so SE must inherit its basic implied assumption, i.e., quantum is an ideal global wave, or de Broglie matter-wave is an ideal global wave. Please note that the concept of matter-wave is an abstract one. There is no detailed physical information about the wave, such as whether it is a shear one or a longitudinal one, scalar one or vector one, with or without a medium, etc. It is a generalized or abstract wave. This abstractness will affect the interpretation of the wave function later.

3.2 SE is an identical relation

The derivation also tells us SE is not an ordinary equation that only holds when certain conditions are met (with specific variable values). It is the operator form of energy definition Eq. (3.3). It describes the system in the dynamical waveform.

It is an **identical relation** instead of an ordinary equation. The standard method of equation solving does not work for SE, for it holds all the time.

3.3 Solving SE needs additional boundary condition

In the practice of solving SE, we **choose** the modes which comply with specific boundary conditions, not find the variables that balance between the two sides of the equation, as in standard equation solving.

The critical factor of the boundary condition is absent in the equation (identical relation) alone. In "solving" SE, we have first to determine the boundary conditions, such as periodicity (as in angular momentum case) or vanishing at specific points. The chosen modes are eigenmodes.

Take the simplest infinitely deep square potential well as an example. Suppose a particle (matter, quantum) is in a one-dimensional infinitely deep square potential well:

$$V(x) = \begin{cases} \infty & x < -a \\ 0 & -a \le x \le a \\ \infty & x > a \end{cases} \tag{3.5}$$

The SE itself does not give any restrictions on the quantum wave function, but we believe from a physical point of view that there should be no wave function at the infinitely high potential, so the wave function at the potential barrier must be 0. This requirement limits the wavelength l of the quantum wave, requiring it to be only an integer fraction of the well width $2a$. Due to the corresponding relationship between wavelength and frequency, the frequency of the quantum wave, corresponding to energy, is now discrete (quantization). We should note that the quantization here is the quantization of energy levels, not the quantum energy because quantum can be any combination of various components in any ratio (superposition).

3.4 SE is a diffusion equation in form

We write down the one-dimensional stationary SE,

$$E\psi = \frac{-\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2} + V(x)\psi \tag{3.6}$$

And the one-dimensional time-dependent Schrödinger equation:

$$i\hbar\frac{\partial\psi}{\partial t} = \frac{-\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2} + V(x)\psi \tag{3.7}$$

Usually, we say that SE is a wave equation, but a wave equation should look like (one-dimension):

$$\frac{\partial^2 u}{\partial t^2} = c^2\frac{\partial^2 u}{\partial x^2} \tag{3.8}$$

u is the wave function, and c is the propagation speed of the wave.

If we ignore the term with V (in free space), the forms of SE Eq. (3.7) and wave equation Eq. (3.8) are different. SE has only first-order time differentiation of the wave function, and the wave equation has second-order time differentiation.

In form, SE is a diffusion equation:

$$\frac{\partial\phi(x,t)}{\partial t} = D\frac{\partial^2\phi(x,t)}{\partial x^2} \tag{3.9}$$

D is the diffusion coefficient.

From the perspective of mathematic equation form, the physical meaning of SE is not a wave equation but a diffusion equation with an imaginary diffusion coefficient of $\frac{i\hbar}{2m}$.

The physical meaning of the diffusion equation is straightforward. Take thermal diffusion as an example. Once a heat source appears in a medium, the heat will diffuse to the entire medium.

The solution to the stationary SE is a fixed probability distribution without diffusion. The time-dependent SE is usually treated with adiabatic approximation (Bohm, 1951), or as a stationary SE.

What does an imaginary diffusion coefficient mean? The presence of imaginary coefficients in a quadratic differential equation usually means fluctuations.

Therefore, we should take the SE as a wave diffusion equation. What is the physical meaning of the solution to the equation? Since its distribution is no longer changing, we can take it as the final state of the matter-wave after sufficient diffusion. Furthermore, the non-relativistic nature of SE means that the propagation speed of the action is infinite. That is, complete diffusion does not take time.

Waves are coherent (Born, 1926), and coherent waves interfere. Therefore, only those with favorable coherence (interference) conditions (frequency) stay if waves sufficiently diffuse.

Therefore, the solution of the SE is the set of all coherence favorable waves in the system, namely, eigenstates or the advantageous vibration modes. Moreover, they must reflect all the spatial properties of the system, such as boundary conditions, symmetry, the nature of the potential, etc.

The above arguments are speculations, not proofs.

3.5 SE is global and an approximation

However, even if we discard the above discussion entirely, the eigenstate argument is still valid. The solution of SE (quantum wave function) reflects the global nature of space and its potential. The wave nature comes from the assumption of the de Broglie matter-wave. The above conjecture and comparison are just for a more intuitive understanding of the physical meaning of the SE.

Due to the equivalence of matrix mechanics and wave dynamics (Schrödinger, 1926b), we can infer that all quantum states are global states, non-relativistic, and have an infinitely fast propagation speed. However, these are only mathematical properties embedded in the formulation, not actual physical ones.

We should also note that the above discussion refers to ideal conditions. All solutions are distributed in the entire space (except for infinitely deep potential wells, which do not exist).

To sum up, the solution to Schrödinger's equation is a set of waves. They are ideal, global, non-relativistic, preferable modes (eigenmodes). By definition, these are also the properties of a quantum.

For comparison, we can discuss the general solutions to time-dependent differential equations. We need initial values and boundary conditions, and the solution is a set of equations that change with time. We can not analytically solve the equation in a real-world system in normal circumstances. Instead, we have to use numerical methods, such as in the computation of fluids, plasmas, electromagnetic fields, etc.

However, we hardly use numerical methods to solve SE. Because, in physics, the problem to be solved is not an initial value problem. Moreover, the spatial and temporal discretization in the numerical calculation means the value on each grid point at the next moment depends only on a certain number of surrounding grids at the previous moment (not including implicit methods). The treatment is only valid for local interactions, but SE is global. Therefore, for eigenvalue problems, numerical methods are very unreliable. Instead, we usually use the matrix method to solve the eigenvalue problem of quantum mechanics (Heisenberg and Jordan, 1926), which is inherently global.

In principle, to calculate the behavior of any single particle or multiple particles of a quantum system, SE requires the knowledge of the state of the entire universe because it is a part of the potential. It is impossible to calculate a real-world system with the extra particle identicalness complications, which is non-relativistic (instantaneously connected), many-body, and ever-changing.

Quantum is not a particle. Non-relativistic QM does not answer what a particle is. It only gives the global properties of any matter-wave in different situations.

Due to the complexity of the real-world system discussed in the previous chapter, we can also say that the solution to SE is the non-relativistic ideal limit of the system.

The non-relativistic approximation of the SE shows that the quantum wave fills the entire space, and the instantaneous response to any change in the whole space is equivalent to assuming that the propagation speed of the wave is infinite. The propagation speed of the interaction is also infinite. For the actual quantum system, it means that the speed of light is infinite. That is, the propagation speed of electromagnetic action is infinite.

3.6 Implications of SE's globality

No one denies the globality of QM formulation nor discusses it in depth. The property has profound significance in the understanding of QM behaviors. However, CI did not discuss the consequences of quantum globality. On the contrary, CI tried to resolve the contradiction with common sense and classical theory with wave-particle duality, correspondence principle, measurement collapse, etc.

The globality of QM has the following significant consequences:

- **Quantization**:

  Quantization is the result of the global nature of SE. Because the global mode, the eigenstate of the bound system, is discrete, the physical quantities of each state, such as energy, angular momentum, etc., are discrete and do not change continuously. In the case of atoms and molecules, they appear as discrete energy levels and spectral lines. Eigenstate discreteness is also the actual reason for Planck's energy quantization (Planck and Dtsch., 1900), and no assumption is needed.

- **Global coherence**:

  All quanta are globally and ideally coherent. If appropriately arranged, they can form intricate interference patterns, as in the quantum eraser experiments (Ma, Kofler, Qarry, Tetik, Scheidl, and et al., 2013; Wimmel, 1992). Coherence, or correlation, appears to be instantaneous and superluminal (quantum non-locality).

- **Global causality**:

  In QM, we can only discuss causality from a global perspective, contrary to local causality provided by the special theory of relativity