A Guide to Practical Seismology

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Cambridge Scholars
Publishing

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This book first published 2023

Cambridge Scholars Publishing

Lady Stephenson Library, Newcastle upon Tyne, NE6 2PA, UK

British Library Cataloguing in Publication Data
A catalogue record for this book is available from the British Library

ISBN (10): 1-5275-9036-4
ISBN (13): 978-1-5275-9036-6

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1 Introduction

1.1 The Earth

The Earth is approximately spherical, with a mean radius $R = 6370km$, a very small flattening $(+7/-15km)$, mass $\simeq 6 \times 10^{24}kg$, and an average density $5.5g/cm^3$; the law of gravitational attraction is $\mathbf{F} = GmM\mathbf{r}/r^3$, where F is the force directed along the separation distance $\mathbf{r}$ between two point bodies with mass m and M; and $G = 6.67 \times 10^{-8}cm^3/g \cdot s^2$ is the gravitation constant.

Little is known about Earth's interior. The drilling down into the earth reaches at most $10-15km$. It is accepted that the Earth consists of several shells. First, at the surface, there is a solid crust, extending down to approximately $70km$, on average; locally it may have $5km$ thickness. Down to approximately $3000km$ an extremely viscous mantle exists. The next $2000km$ down to the centre are occupied by a liquid outer core. Finally, a solid inner core exists at the centre. The viscosity of the mantle is $10^{22} - 10^{25}g/cm \cdot s$; for comparison, water has $10^{-2}g/cm \cdot s$. The physical properties are discontinuous at the boundaries of these layers, like the "Moho" discontinuity (named after Mohorovicic) between crust and mantle, or the Gutenberg discontinuity between mantle and the liquid core, or the Bullen (or Lehmann) discontinuity between the inner core and the outer core. Changes in propagation of the elastic (seismic) waves have been measured and have indicated such discontinuities. The main chemical elements in the Earth's shells are Fe, O, Si, Mg, S, Ni, Co, Al. The crust is made mainly of silicon dioxide and aluminium oxide. The crust density is $3g/cm^3$; the inner core has probably the density $13g/cm^3$. The crust and the upper part of the mantle are called lithosphere; tectonic plates are located there, in slow motion. This motion is known as the continental drift. The largest rate of the continental drift seems to be $2.5cm/year$ (separation of the Americas from Europe and Africa).[1] The Earth's crustal movements are measured today by satellites in the Global Positioning System (GPS). Very likely, the earthquakes, volcanoes and mountains are produced at plate boundaries. The gravitational acceleration at the Earth's surface $g = 9.8m/s^2$ is preserved down to $3000km$, where it decreases appreciably. Earth's magnetic field is produced probably by convection and motion of electrical charges in the liquid outer core; the Earth's temperature is due probably to radioactive decays. In the inner core the temperature is probably $6000K$ and the pressure is $3.5 \times 10^{12} dyn/cm^2$.

A great deal of physical phenomena can be observed and even quantitatively measured, related to the internal motion in the Earth. Among these are heat flow, quasi-static displacement (the motion of the tectonic plates), strain, variations in gravity, electromagnetic phenomena; and, of course, seismic waves. The range of these variables is huge. Explosive charges are detected from $1g$ to $10^9 kg$; ground displacements are measured from 1μ to tens of meters for the slip of a major fault during an earthquake. Earthquakes vary in intensity over more than 18 orders in energy (one of the greatest earthquake was the Chilean earthquake of 1960, May 22). Seismic networks vary from tens of meters for an engineering foundation survey to $10^4 km$ for the global array of seismological observatories.

1.2 Seismic sources and waves

The Earth is a deformable body, which may bear local elastic movements, like static deformations and elastic waves. Among these, the greatest are the earthquakes. The Earth may be viewed as an elastic body, in the first approximation an infinite homogeneous and isotropic elastic medium; however, the effects of the earthquakes are felt on its surface, so the surface should be included.

The mechanics of deformable bodies and the theory of elasticity appeared gradually, over more than two hundred years. About 1660

[1] A. Wegener, "Die Herausbildung der Grossformen der Erdrinde (Kontinente und Ozeane) auf geophysikalischer Grundlage", Petermanns Geographische Mitteilungen **63** 185, 253, 305 (1912); *Die Entstehung der Kontinente und Ozeane*, Vieweg & Sohn, Braunschweig (1929).

Hooke determined the proportionality of the force with the deformation: *ut tensio, sic vis*. In 1821 Navier established the equilibrium of the elastic bodies and their vibrations; Young and Fresnel showed the polarization of the waves, in relation to the transverse polarization of light; in 1822 Cauchy introduced the strain and the stress tensors; then, Poisson determined the compression and the shear elastic waves, and Green introduced the strain-energy function; Kelvin computed the static deformation produced by a localized force and Stokes derived the elastic waves from a localized force in an infinite elastic medium. Elastic waves and vibrations were studied intensively by Rayleigh, Lamb and Love at the end of the 19th century and the beginning of the 20th century. Non-linear elasticity (also called "finite" elasticity), or the mechanics of deformable visco-elastic media, or of micro-structured media are various generalizations.

It is widely accepted that the earthquakes are produced by a sudden release of the elastic energy built up locally at the boundary of two or more tectonic plates; such an interface of tectonic plates, where a rupture in the material may occur, is a fault. The spontaneous slip occurring in a fault, or explosions, is a seismic source. The force acting in a faulting source is usually related to the fault slip over a finite faulting area. For a "volume" source, like those associated with explosions, the force is related to the dilatational strain. The focus of an earthquake is localized in a small volume, of various shapes which, however, are irrelevant as long as the focal volume may be viewed as being concentrated in a point. In the first approximation, the fault slip, occurring in the focus, is characterized by a direction; the pressure, caused by an explosion for instance, is the (uniform) force per unit area on the surface enclosing a localized, small volume. The focus is active a certain duration of time (which often may be taken as an impulse-like duration). In general, a faulting slip generates tensorial forces; a faulting source is characterized by a tensor; this is the tensor of the seismic moment. The "volume" sources correspond to an isotropic tensor of seismic moment (isotropic sources). As long as the material remains elastic the seismic forces generate a slow movement of the tectonic plates; such a quasi-static deformation relieves the stress and diminishes the probability of an earthquake. If the material yields, and a sudden rupture appears, the seismic forces generate a shock and cause an earthquake. Hence, monitoring the small displacement of the tectonic plates, especially in regions prone to earthquakes, may give an indication about the likelihood of an earthquake.

The rock fracture and the faulting mechanism for seismic sources (located from a few kilometers beneath the Earth's surface down to $700km$) have been accepted gradually, especially after the big earthquakes of Mino-Owari (Japan), 1891 and San Francisco, 1906. In a big earthquake soil displacement as large as a few meters may appear, both horizontally and vertically, along distances as large as tens to hundreds of kilometers,[2] and accelerations may exceed the gravitational acceleration; sometimes, the soil displacement is permanent. An extensive phenomenology of the earthquakes was given by Richter.[3]

Although Earth is not a perfect elastic body, it is still approximated by a simple, homogeneous and isotropic elastic solid, with two elastic moduli: the Lame coefficients λ and μ, or the Young modulus E and the Poisson ratio σ.[4] In such an (infinite) elastic solid two kinds of elastic waves can be propagated: longitudinal waves, associated with the compressibility of the solid, and transverse waves, associated with the shear elastic properties of the solid. The longitudinal waves propagate faster; they are called P waves ("primary", compressional waves); their velocity (in the crust) is approximately $7km/s$; the transverse waves are slower, they are called S waves ("secondary", shear waves); their velocity in the crust is approximately $3km/s$.[5] In general, the velocity of the elastic wave seems to increase with the increasing depth in the earth.

The problem of propagation of the seismic waves is complicated by the presence of the Earth's surface, which suggests to approximate the Earth by an elastic half-space (we may neglect in the first approximation the Earth's surface curvature). The equations of motion of the linear elasticity are second-order differential equations in

[2]H. F. Reid, *Mechanics of the Earthquake, The California Earthquake of April 18, 1906*, vol. 2, Carnegie Institution, Washington (1910).

[3]C. F. Richter, *Elementary Seismology*, Freeman, San Francisco, CA (1958).

[4]We may take as mean values for Earth $E \simeq 10^{11} dyn/cm^2$, $\mu = E/2(1+\sigma) \simeq 10^{11} dyn/cm^2$, $\lambda = E\sigma/(1-2\sigma)(1+\sigma) \simeq 10^{11} dyn/cm^2$ (Poisson ratio $\sigma \simeq 0.2$).

[5]The notations P and S seem to originate historically in "primary" and "secondary" waves. Both will be called here primary waves.

variables time and position (elastic waves equation, known also as the Navier-Cauchy equation). Such an equation is amenable to two distinct approaches. First, we may consider a source appearing at a certain moment of time, which produces waves. Before, there is no motion. This is the propagating-wave approach, governed by the causality principle: the waves are produced only by sources acting in the past. The waves propagate in the future. The wave source lasts a finite time, usually a short time; in this case it produces localized waves, like the P and S seismic waves, which have a spherical-shell structure. This is the case of earthquakes. Once arrived at Earth's surface, these seismic waves (primary seismic waves) produce surface sources of secondary waves, according to Huygens principle, propagating on the surface, which gives the seismic main shock (actually, two main shocks, corresponding to the two distinct velocities of the seismic waves). The seismic main shock is a delocalized wave propagating back in the Earth. On the surface it looks like a propagating wall, behind the primary waves, with a long tail (actually two walls, corresponding to the two P and S waves, for distinct components of the elastic displacement). Despite having a long tail, the main shock is a wave, propagating from a certain instant towards the future. This is a transient regime, seen and felt in earthquakes. The inner layers of the Earth may cause dispersion of the elastic waves.[6] After a while, the seismic waves suffer multiple reflections on the spherical Earth's surface, their presence is continuous on the whole Earth's surface, and they produce free oscillations of the Earth (eigenoscillations, eigenvibrations, normal modes); the seismic source ceased its action since long. The frequencies of the P and S seismic waves are, mainly, of the order $1s^{-1}$ (it seems that the periods of the seismic waves are in the range $0.1s$-$10s$, though these are only orientative figures); their wavelength is longer than the dimensions of the focal region. The free oscillations of the Earth may have much lower frequencies.

The second approach consists in viewing a continuous source, whose effects (waves) act continuously upon the whole Earth's surface. They should obey boundary conditions on the surface. The time is indefinite in this approach, the role of the causality principle is played

[6] A. E. H. Love, *Some Problems of Geodynamics*, Cambridge University Press, London (1911).

now by the boundary conditions. This is the vibration approach. In vibration, the waves propagate both in the future and in the past, their superposition gives a vibration. A source may produce forced vibrations, in the absence of a source the vibrations are free (free oscillations, eigenoscillations). The vibration regime is a stationary regime, distinct from the propagating-wave transient regime. In 1885 Rayleigh discovered that an important contribution to the vibrations of a homogeneous and isotropic elastic half-space with a free plane surface is brought by damped waves, called Rayleigh's surface waves (actually vibrations).[7] Since they last long, over large distances (as any vibration), it was tempting to associate them with the seismic main shock. The Rayleigh's surface waves propagate as plane waves along the surface and are damped vibrations along the direction perpendicular to the surface; in another nomenclature they may be called guided waves.

The recording of the seismic waves in seismograms shows approximately, very approximately, a general, qualitative picture of P and S elastic waves and main shock (shocks). The first attempt at constructing a theoretical seismogram was done by Lamb in 1904 for a seismic source on the surface of a homogeneous and isotropic elastic half-space or buried in such a half-space;[8] Lamb's results consist of a sequence of three pulses, which Lamb associated to a preliminary feeble tremor (say, P, S waves) and Rayleigh surface "waves", according to their arrival times (the surface waves are the slowest). In fact, Lamb's solution is for a vibration problem, not for a propagating wave problem. For a temporal-impulse source, Rayleigh's and Lamb's solution (though inapplicable) extends over the whole free surface, much before the arrival of a main shock, albeit exhibiting a propagating double-wall structure.[9] Even the seismic main shock, though extended over the surface, is not a vibration, because its source, the source of secondary waves, lies just on the surface, and boundary conditions are meaningless in this case. On the other hand, a construction

[7]Lord Rayleigh, "On waves propagated along the plane surface of an elastic solid", Proc. London Math. Soc. **17** 4 (1885).

[8]H. Lamb, "On the propagation of tremors over the surface of an elastic solid", Phil. Trans. Roy. Soc. (London) **A203** 1 (1904).

[9]B. F. Apostol, "On the Lamb problem: forced vibrations in a homogeneous and isotropic elastic half-space", Arch. Appl. Mech. **90** 2335 (2020).

built on Earth's surface, under the action of the seismic waves may suffer vibrations, of course. The first seismogram was recorded in the early 1880s. P, S and a main shock (interpreted as surface waves) have been first recognized on a seismogram by Oldham in 1900.[10] Reflections of the original seismic pulses in Earth's surface layers may generate long-lasting oscillations; for a long time Earth vibrates and oscillates (rather than "radiates"), as Jeffreys discussed in 1931;[11] seismic waves may be localized in Earth's surface layers. In general, the scattering of the seismic waves by inhomogeneities makes them to last long. The seismograms exhibit a characteristic, long tail (coda).

One of the main problems of the Seismology is to understand the seismograms, *i.e.* the seismic movement recorded at Earth's surface. This is known as the seismological problem. Typically, any seismogram exhibits a preliminary tremor of feeble movement, which consists of spherical-shell P- and S-seismic waves, followed by a main shock (or two main shocks); the main shock finally subsides slowly with a long seismic tail. The recorded pattern exhibits many oscillations. Spherical waves are generated by seismic sources localized in space and time. These waves interact with the Earth's surface and produce additional sources moving on the Earth's surface. These surface sources generate secondary waves, which propagate in the whole Earth; on the surface they cause the main shock and its long tail. The oscillations are probably due to the internal structure of the earthquake focus, which may include successive and adjacent point ruptures (the structure factor of the earthquake focal region); also, inhomogenities of the Earth, multiple reflections and the seismographs' characteristics may play a role in these oscillations.

1.3 Empirical laws

The energy released by an earthquake was, and still is, estimated by the damage produced by the seismic waves at Earth's surface, in

[10] R. D. Oldham, *Report on the Great Earthquake of 12th June, 1897*, Geol. Surv. India Memoir 29 (1899); "On the propagation of earthquake motion to long distances", Trans. Phil. Roy. Soc. London **A194** 135 (1900).

[11] H. Jeffreys, "On the cause of oscillatory movement in seismograms", Monthly Notices of the Royal Astron. Soc., Geophys. Suppl. **2** 407 (1931).

the region affected by the earthquake. The highest known value is of the order $10^{30}erg$, for the 1960 Chilean earthquake, or the 1964 Alaskan earthquake. The smallest value is about $10^{12}erg$ for micro-earthquakes; $10^{5}erg$ corresponds to micro-fractures in laboratory experiments on loaded rock samples. For such large variations a logarithmic scale is convenient. The earthquake magnitude M appeared this way, defined by

$$E/E_0 = e^{bM} \quad , \tag{1.1}$$

where E_0 is a threshold energy and b is a constant, chosen by convention $b = 3.45$.[12] Originally, the law was written with powers of ten, where $b = 3/2$. By another convention, the parameter E_0, measured in erg, is given by $\lg E_0 = 15.65$ (decimal logarithm). The logarithmic form of this law is also known as the Gutenberg-Richter law.[13] Later on, the earthquake energy was associated with the tensor of the seismic moment, and a similar logarithmic law was introduced for the magnitude of this tensor, which defines a so-called earthquake moment magnitude. This law is called the Hanks-Kanamori law.[14]

It was shown empirically that the number of earthquakes with magnitude greater than M which appear in a given region in a given time duration obeys a logarithmic law

$$\ln N = const - \beta M \quad , \tag{1.2}$$

where β is a constant which depends on the data set (like *const* too). A reference value $\beta = 2.3$ is accepted (in decimal logarithms $\beta = 1$). This is a statistical law. It is called also the Gutenberg-Richter law (known also as the excedence, or cumulative law).

It was noticed that a big earthquake is often preceded and succeeded by many smaller earthquakes, which appear in the same seismic region

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[12] T. Utsu and A. Seiki, "A relation between the area of aftershock region and the energy of the mainshock" (in Japanese), J. Seism. Soc. Japan **7** 233 (1955); T. Utsu, "Aftershocks and earthquake statistics (I): some parameters which characterize an aftershock sequence and their interaction", J. Faculty of Sciences, Hokkaido Univ., Ser. VII (Geophysics) **3** 129 (1969).

[13] B. Gutenberg and C. Richter, "Frequency of earthquakes in California", Bull. Seism. Soc. Am. **34** 185 (1944); "Magnitude and energy of earthquakes", Annali di Geofisica **9** 1 (1956) (Ann. Geophys. **53** 7 (2010)).

[14] H. Kanamori, "The energy release in earthquakes", J. Geophys. Res. **82** 2981 (1977); T. C. Hanks and H. Kanamori, "A moment magnitude scale", J. Geophys. Res. **84** 2348 (1979).

and in a reasonably short time interval; they are called foreshocks and aftershocks, respectively. The time distribution of these accompanying earthquakes is given approximately by

$$\frac{\Delta N}{\Delta t} \sim \frac{1}{const + t} \ , \tag{1.3}$$

where ΔN is the number of earthquakes which appear in the time interval Δt measured with respect to the main shock. This is known as Omori's law.[15]

Finally, it seems that the greatest aftershock of a main shock has a magnitude smaller by $\Delta M = 1.2$ than the main shock. This is known as Bath's law.[16]

All that we know about earthquakes reduces practically to these empirical, disparate laws. If we add the lack of knowledge of the P and S seismic waves and the main shock, we can see that we do not know much about earthquakes. It is claimed that the tensor of the seismic moment is related to the main shock, the later associated with surface waves; and, from measurements of these waves we may have information about the seismic moment and the magnitude of the earthquakes (through the Hanks-Kanamori law). Also, it is claimed that this knowledge is incorporated in numerical codes, released by various agencies, to be used for the determination of the seismic moment, the magnitude and, possibly, other earthquakes parameters. However, this knowledge is not in the public domain and, consequently, cannot be checked.

[15] F. Omori, "On the after-shocks of earthquakes", J. Coll. Sci. Imper. Univ. Tokyo **7** 111 (1894).

[16] M. Bath, "Lateral inhomogeneities of the upper mantle", Tectonophysics **2** 483 (1965); C. F. Richter, *Elementary Seismology*, Freeman, San Francisco, CA (1958) p. 69.

1.4 Seismology

Recently, basic results have been obtained in Seismology,[17] which are described briefly below.

The law of energy accumulation with increasing time in a pointlike focus was obtained by using the continuity equation and the energy conservation. The geometric-growth parameter r has been introduced ($1/3 < r < 1$), which makes the difference between $\beta = br$ and b. Also, besides the energy threshold E_0, the basic time threshold t_0 has been introduced, which gives the seismicity rate $1/t_0$.

Based on this law, the time and energy probability distributions have been derived, as well as the standard Gutenberg-Richter magnitude distributions. This way, the background seismicity of regular earthquakes has been defined and Omori's law of conditional probability has been derived. These laws have been applied to the the estimation of the mean recurrence time of earthquakes and to the analysis of next-earthquakes distribution, both results of practical relevance.

The law of energy accumulation and the time and magnitude distributions have been used to derive the bivariate distributions, which account for the earthquake correlations in the foreshock-main shock-aftershock sequences. Time-magnitude, dynamical and statistical correlations have been highlighted, and correlation-modified Gutenberg-Richter distributions have been derived. This way, Bath's law has been explained and a procedure was established for estimating the occurrence time of a main shock, by using the analysis of the foreshocks. In the vicinity of a main shock the parameter β decreases, while the number of aftershocks and the parameter β increase after a main shock, as a result of the change in the seismicity conditions brought about by the main shock.

The tensorial force acting in a pointlike focus has been established and the notion of the elementary earthquake (temporal-impulse force) has been introduced. This force is governed by the tensor M_{ij} of the seismic moment. The elastic wave equation has been solved with this

[17] B. F. Apostol, *The Theory of Earthquakes*, Cambridge International Science Publishers, Cambridge (2017); *Introduction to the Theory of Earthquakes*, Cambridge International Science Publishers, Cambridge (2017); *Seismology*, Nova, NY (2020.

force, as well as the elastic equilibrium equation (Navier-Cauchy equations). This way, the P and S seismic waves have been derived, the transient regime of the seismic waves has been established, and the static deformations at Earth's surface have been computed. A regularization method has been introduced for the solutions of the elastic wave equation and a new method has been introduced for computing static deformations of an elastic half-space with general forces.

The main shock produced by the seismic waves, especially on Earth's surface, has been computed, and its singular wavefront has been derived. Thus, together with the derivation of the P and S seismic waves, the structure of the seismograms has been explained. This is known as the seismological (or Lamb's) problem. The identification of the transient regime was instrumental in solving this problem.

The above results have been applied to the seismological inverse problem, *i.e.* the derivation of the tensor of the seismic moment. The seismic moment was derived from the P and S seismic waves measured at Earth's surface, by using the Kostrov representation and the energy conservation. On this occasion, all the parameters of the seismic source were derived from empirical measurements of the P and S waves, like the duration of the seismic activity in the earthquake focus, the dimension of the focus, the focus strain and its rate, the orientation of the fault and the slip along the fault. The relation $\left(M_{ij}^2\right)^{1/2} = 2\sqrt{2}E$ between the magnitude of the seismic moment and the energy (E) has been established. Also, the results have been applied to explosions, which have an isotropic tensor of the seismic moment. It was shown that a hybrid focal mechanism, which would imply a shear faulting and an isotropic dilatation (or compression) is impossible. A procedure has also been devised for getting the seismic moment from the (very small) static displacements measured (theoretically) in the epicentral region.

Finally, the effect of the seismic movements on the constructed structures on Earth's surface has been investigated by using the models of the embedded bar, the buried bar and coupled oscillators (and bars). Amplification factors have been derived and the risk brought by soft inclusions, and, in general, by inhomogeneities in elastic structures was highlighted. A new method for computing the elastic vibrations of a half-space has been introduced and the difference between propagating waves and vibrations was emphasized; also, two-dimensional related problems were solved.

Most of these subjects can also be found in a previously published book.[18] Apart from new, original points, the present book emphasizes the practical side of such problems of seismological interest.

1.5 Description of the book

The first part of the book is devoted to statistical Seismology (the first four chapters). The geometric-growth model for energy accumulation in the focus is introduced, the time distribution of earthquakes is established and the standard Gutenberg-Richter earthquake distribution in magnitude is deduced. Conditional probabilities are described and the Omori distribution is derived. Aplications to Vrancea earthquakes are presented. It is shown that the background seismicity (consisting of regular earthquakes) of a region over a long period of time is characterized by two parameters: the slope of the Gutenberg-Richter distribution and the seismicity rate. The recurrence time (periodicity problem) is governed by these parameters. Further on, the next-earthquake distributions are introduced, and their practical relevance is emphasized. A main subject in statistical Seismology is the earthquake correlations, which characterize the distribution of the foreshocks and the aftershocks (accompanying seismic events). The pair correlations are established, the roll-off effect is explained and the nature and characteristics of the accompanying seismic events are presented. The pair correlations explain the deviations from the standard Gutenberg-Richter distribution. On this occasion the Bath's law is explained. Short sequences of correlated foreshocks may be used to make a short-term prediction of a main shock, including its occurrence moment and magnitude. A few examples of application of this procedure are given. Also, in this part the problem of the statistical equilibrium of a seismic zone is discussed, by means of the earthquake entropy. It is shown that the seismic activity is a non-equilibrium process, where the steadily decreasing entropy is interrupted from time to time by abrupt increases, due to big earthquakes.

[18] B. F. Apostol, *Seismology*, Nova, NY (2020).

The second part of the book deals with seismic waves, seismic moment and the static deformations (three chapters). First, the tensorial seismic point force is introduced. This is a temporal-impulse force, localized in space and governed by the tensor of the seismic moment. It is shown how the P and S seismic waves can be derived from this force, and how these primary waves generate on Earth's surface sources of secondary waves, which generate the main shock(s). The primary seismic waves are scissor-like spherical shells, and the main shock comes out with an abrupt wall and a long tail, propagating on the surface behind the primary waves. This way, the seismological problem is solved. A complete solution would require the determination of the seismic-moment tensor and the other parameters of the earthquakes by measurements performed on Earth's surface. We show that the amplitude of the P and S seismic waves provides a means of determining this tensor. The determination procedure is conducted in a consistently covariant way. It gives access to the earthquake energy and magnitude, as well as to the dimension of the focus, the duration of the seismic activity in the focal region and the orientation of the fault. The introduction of the seismic tensorial force raises another problem, besides the seismological problem: the computation of the static deformations generated on Earth's surface. The result of these computations is presented and discussed in this book. Moreover, the periodic small discharge of the seismic stress generates small deformations in the epicentral zone. We show, on one side, how to estimate an average seismic moment and the depth of the focus by measuring such quasi-static deformations; and, on the other side, we show that a continuous monitoring of the quasi-static epicentral deformations may give information about a possible evolution of the seismic activity, because, after periods of silence we may expect a burst of seismic activity.

The next three chapters of the book deal with local seismic effects. An important problem in Seismology is how to secure the buildings erected on Earth's surface against the destructive action of the earthquakes. We show that the buildings may be viewed as vibrating bars, which, under the seismic action may resonate; also, sub-surface inhomogeneities may behave as resonating embedded (buried) bars. In both cases we get local amplification factors, evaluated in the book.

The estimation of the local effects requires the understanding of the relevance of the site spectral response, which is currently measured on Earth's surface. We discuss the information the site response may give, in the complex context of the presence of the inhomogeneities, different local velocities of the elastic waves, or different wave polarizations. A special chapter is devoted to the interaction of a harmonic-oscillator model with an elastic wave, the associated amplification factors, the role of damping, the different behaviour under seismic shocks and periodic elastic waves.

Hitherto, besides giving theoretical tools, sometimes in great detail, necessary for understanding the seismic phenomenon, we focused on practical procedures relevant for Seismology. The book describes in detail activities (tasks) which can be undertaken by anybody in order to get knowledge about earthquakes. For instance, by updating periodically the background earthquake activity, we may get information about the recurrence time of the big earthquakes; the next-earthquake distribution may answer our current question: "what may we expect after an important earthquake?"; by fitting (daily, sometimes hourly) the correlated foreshock sequences we may predict, on a very short term, a main seismic shock; by updating periodically the earthquake entropy we may see if something changed in the seismic non-equilibrium process, so we may get an insight on what we may expect in the future in regard to earthquakes. By using the procedure described in this book we may determine the seismic moment, the magnitude and the characteristics of the fault; or, we may assess the behaviour of the small epicentral seismic discharges. By measuring the local effects, the amplification factors and the site spectral response, we may get information about improving the design of the constructions. By using such practical procedures, the Seismology reveals its practical side, besides its pure-science side, which may become useful in our daily life.

The end part of the book deals with a few more theoretical issues (three chapters). Such issues are not of practical use, but they may help us to better understanding of the problems raised by the seismological studies. The first problem tackled in this context is the problem of the inhomogeneities dispersed naturally in the Earth. We give a detailed presentation of this problem, regarding both bulk and surface inhomogeneities (rough surface). We show that the bulk inhomogeneities may renormalize the velocity of the elastic waves, may produce a slight distortion of the localized seismic waves and may produce dispersion of the waves, as expected. In general, such effects have, relatively, little relevance, though, in some cases, a rough surface may generate the surprising effect of a surface localized wave. The next problem is the vibration problem. In the seismological studies the wave propagation and the elastic vibrations are treated to some extent indistinctly. We introduce a new mathematical technique of treating the elastic vibrations, which serves to distinguish them sharply from propagating waves, and apply this technique to a half-space. In this context we discuss amply the so-called Lamb problem. Further on, we dedicate a rather long space to the elastic vibrations of a sphere, which is an old and important subject in Seismology. We are concerned in particular with the effect of gravitation and rotation and with approximate techniques of estimating vibrations in the case of the Earth, which has a large radius. Finally, we devote the end chapter to a rather academic problem, concerning the seismic phenomenon in two dimensions. Though far from a direct physical relevance, this problem is often discussed because, in some cases, it may look more simple than the three dimensional problem. We compute the seismic waves in two dimensions, the main shock and the vibrations of an elastic half-plane.

We hope, according to the above description, that the book may be useful, not only for an understanding of the seismic phenomena, but, especially, to those who wish to be active in Seismology.

We are very thankful to our colleagues in the Department of Engineering Seismology in the Institute for Earth's Physics and the Department of Theoretical Physics in the Institute for Physics and Nuclear Engineering, both at Magurele, for enlightening discussions, very useful suggestions, pointing out errors and offering corrections (many accepted and included) and a thorough critical reading of the manuscript.

2 Background Earthquakes

2.1 Geometric-growth model

We consider a typical earthquake (which we call a tectonic earthquake), with a small focal region localized deeply in the solid crust of the Earth. Also, we assume that the movement of the tectonic plates (rocks) leads to energy accumulation in this pointlike focus. (For surface earthquakes the focal mechanism may exhibit additional features, for instance a propagating focus. A propagating focus can also be associated with a deep extended shear faulting).[1]

The energy accumulation in the focus is governed by the continuity equation (energy conservation)

$$\frac{\partial E}{\partial t} = -\boldsymbol{v} gradE \ , \tag{2.1}$$

where E is the energy, t denotes the time and $\boldsymbol{v}$ is an (undetermined) accumulation velocity. For a localized focus we may replace the derivatives in equation (2.1) by ratios of finite differences. For instance, we replace $\partial E/\partial x$ by $\Delta E/\Delta x$, for the coordinate x. Moreover, we assume that the energy is zero at the borders of the focus, such that $\Delta E = -E$, where E is the energy in the centre of the focus. Also, we assume a uniform variation of the coordinates of the borders, given by equations of the type $\Delta x = u_x t$, where $\boldsymbol{u}$ is a small (undetermined) displacement velocity of the medium in the focal region. The energy accumulated in the focus is gathered from the outer region of the focus, as expected. We note that the displacement of the rocks in the focal region affects larger zones with increasing time. With these

[1] B. F. Apostol, *Theory of Earthquakes* and *Introduction to the Theory of Earthquakes*, Cambridge International Science Publishing, Cambridge (2017).

assumptions equation (2.1) becomes

$$\frac{\partial E}{\partial t} = \left(\frac{v_x}{u_x} + \frac{v_y}{u_y} + \frac{v_z}{u_z} \right) \frac{E}{t} \ . \tag{2.2}$$

Let us assume an isotropic compression without energy loss; then, the two velocities are equal, $\boldsymbol{v} = \boldsymbol{u}$, and the bracket in equation (2.2) acquires the value 3. In the opposite limit, we assume a one-dimensional compression. In this case the bracket in equation (2.2) is equal to unity. An energy loss may exist in this case, as a consequence of a back-displacement, off the focus along the other two directions, such that the bracket in equation (2.2) may have a value slightly smaller than unity. A similar analysis holds for a two dimensional accumulation process, such that, in general, we may write equation (2.2) as

$$\frac{\partial E}{\partial t} = \frac{1}{r} \frac{E}{t} \ , \tag{2.3}$$

where the parameter r varies in the range (1/3, 1).

The integration of this equation needs a cutoff (threshold) energy and a cutoff (threshold) time. We may imagine that during a short time t_0 a small energy E_0 is accumulated. In the next short interval of time this energy may be lost, by a relaxation of the focal region. Consequently, such processes are always present in a focal region, although they do not lead to an energy accumulation in a focus. We call them fundamental processes (or fundamental earthquakes, or E_0-seismic events). It follows that we must include them in the accumulation process, such that we measure the energy from E_0 and the time from t_0. The integration of equation (2.3) leads to the law of energy accumulation in the focus[2]

$$t/t_0 = (E/E_0)^r \ . \tag{2.4}$$

The time t in this equation is the time needed for accumulating the energy E, which may be released in an earthquake (the accumulation time).

[2] B. F. Apostol, "A Model of Seismic Focus and Related Statistical Distributions of Earthquakes", Phys. Lett. **A357** 462 (2006); "A Model of Seismic Focus and Related Statistical Distributions of Earthquakes", Roum. Reps. Phys. **58** 583 (2006).

2.2 Gutenberg-Richter statistical distributions

Let us assume a time interval $(0, t)$ divided in cells of duration t_0. In each cell we may have an event with energy E_0 (a fundamental process). We view these events as independent events. We have a total number t/t_0 of such cells, such that the probability to have an E_0-event in the interval (t_0, t) is t_0/t. On the other hand, let us assume that $P(t')dt'$ is the probability to have an earthquake with energy $E > E_0$ in the time interval $(t', t' + dt')$. Also, we view these earthquakes as independent events. The probability to have an earthquake with energy $E > E_0$ in the time interval (t_0, t) is $\int_{t_0}^{t} dt' P(t')$. Consequently, we have the equality

$$\frac{t_0}{t} + \int_{t_0}^{t} dt' P(t') = 1 \ , \tag{2.5}$$

which gives, by differentiation, the probability density

$$P(t) = \frac{t_0}{t^2} \ . \tag{2.6}$$

This is a single-event probability distribution of independent events. It is worth noting that $(t_0/t^2)dt$ is the probability to have an earthquake in the interval $(t, t + dt)$, with no other conditions regarding the time before the time moment zero and after the time duration t.

Making use of accumulation equation (2.4), we get from equation (2.6) the energy distribution [3]

$$P(E)dE = \frac{r}{(E/E_0)^{1+r}} \frac{dE}{E_0} \ . \tag{2.7}$$

At this point we may use the exponential law[4] $E/E_0 = e^{bM}$, where

[3] A power law $E^{-\alpha}$ for the energy distribution of earthquakes was suggested as early as 1932 by K. Wadati, "On the frequency distribution of earthquakes", J. Meteorol. Soc. Japan **10** 559 (1932), with an estimated exponent $\alpha = 0.7 - 2.3$.

[4] H. Kanamori, "The energy release in earthquakes", J. Geophys. Res. **82** 2981 (1977); T. C. Hanks and H. Kanamori, "A moment magnitude scale", J. Geophys. Res. **84** 2348 (1979); see also B. Gutenberg and C. Richter, "Frequency of earthquakes in California", Bull. Seism. Soc. Am. **34** 185 (1944); "Magnitude and energy of earthquakes", Annali di Geofisica **9** 1 (1956) (Ann. Geophys. **53** 7 (2010)).

M is the earthquake magnitude and $b = \frac{3}{2} \cdot \ln 10 = 3.45$ ($\ln 10 = 2.3$), by convention. This equation is the definition of the earthquake magnitude. Historically, it was assumed that the energy E released by an earthquake would be distributed, quasi-uniformly, in a region with the volume $\sim R^3$ (a sphere with radius R). According to the exponential formula $E/E_0 = e^{bM}$ we have $R^3 \sim 10^{bM}$ and $R \sim 10^{\frac{1}{3}bM}$, in powers of ten. The area of the surface covering this volume is $S \sim R^2 \sim 10^{\frac{2}{3}bM}$. It was assumed that an estimate of this area could be obtained from the area affected on Earth's surface by an earthquake, including its subsequent companions (aftershocks). By convention, it has been taken $\lg S = M + const$, which leads to $b = \frac{3}{2}$ (in decimal logarithms).[5] It is worth noting that such assumptions imply that an earthquake and its aftershocks (at least those in its immediate temporal neighbourhood) may be viewed as a single seismic event. The estimation of an earthquake energy from the damaged area on the surface of the Earth is still used today.

Making use of $E/E_0 = e^{bM}$ in equation (2.7) we get the (normalized) magnitude distribution

$$P(M)dM = \beta e^{-\beta M}dM \ , \tag{2.8}$$

where $\beta = br$. In decimal logarithms, $P(M) = \frac{3}{2}r \cdot 10^{-\frac{3}{2}rM}$, where $0.5 < \frac{3}{2}r < 1.5$ (for $1/3 < r < 1$). Usually, the mean value $\frac{3}{2}r = 1$ ($\beta = 2.3$) is used as a reference value, corresponding to $r = 2/3$.[6] We note that the magnitude zero corresponds to energy E_0, such that an E_0-event means, in fact, the absence of any earthquake.

The magnitude distribution can be used to analyze the empirical dis-

[5] T. Utsu and A. Seiki, "A relation between the area of aftershock region and the energy of the mainshock" (in Japanese), J. Seism. Soc. Japan **7** 233 (1955); T. Utsu, "Aftershocks and earthquake statistics (I): some parameters which characterize an aftershock sequence and their interaction", J. Faculty of Sciences, Hokkaido Univ., Ser. VII (Geophysics) **3** 129 (1969).

[6] S. Stein and M. Wysession, *An Introduction to Seismology, Earthquakes, and Earth Structure*, Blackwell, NY (2003); A. Udias, *Principles of Seismology*, Cambridge University Press, NY (1999); T. Lay and T. C. Wallace, *Modern Global Seismology*, Academic, San Diego, CA (1995); C. Froelich and S. D. Davis, *Teleseismic b values; or much ado about* 1.0, J. Geophys. Res. **98** 631 (1993).