

The Tribological and Structural Behavior of Untextured and Textured Plain Bearings

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By

Nadia Bendaoud and Mehala Kadda

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FOREWORD

Hydrodynamic bearings and aerodynamic bearings are components which ensure the guiding of rotating machines, such as turbines and reactors. This equipment operates under very severe operating conditions, with a high rotational speed and high radial load. In order to improve the hydrodynamic performance of these rotating machines, manufacturers specializing in the manufacture of hydrodynamic plain bearings have designed a bearing model with its textured interior surface. This book provides a numerical analysis carried out to observe the effect of a turbulent fluid flow in a non-textured and textured plain bearing and to see the improvement in hydrodynamic and tribological performance on both un-textured and textured surfaces of plain bearings, under severe operating parameters. It also presents an analysis of the structural behavior of a hydrodynamic and aerodynamic bearing with a non-textured and a textured surface.

NOTATIONS

B	: Terme source
N	: Vitesse de rotation [tr/min]
$\vec{r}$	: Vecteur de position [m]
U	: Vitesse périphérique [m/s]
u, v, w	: Vitesses suivants l'axe x, y, z [m/s]
ε_t	: Déformation totale [m]
ϕ	: Facteur de débit[°]
ν	: Coefficient de Poisson
c_s	: Chaleur spécifique [J/kg.k]
e	: Excentricité [m]
h	: Epaisseur du film [m]
k	: Conductivité thermique [w/m.K]
p	: Pression [MPa]
t	: Temps [s]
u_a	: Déformation thermique de l'arbre [m]
u_c	: Déformation thermique du coussinet [m]
x, y	: Coordonnés Cartesien, $x = R_a \theta$
C	: Jeu radiale [m]
L	: Longueur du palier [m]
O_a	: Centre de l'arbre
O_c	: Centre du coussinet
R_a	: Rayon de l'arbre [m]
R_c	: Rayon du coussinet [m]
T	: Température [°C]
T_0	: Temperature initiale [°C]
W	: Charge radiale [N]
α	: Coefficient de dilatation thermique [1/K]
δ	: Composant thermo-élastique [m]
ε	: Excentricité relative $\varepsilon = e / C$
μ	: Viscosité dynamique [Pa.s]
μ_0	: Viscosité dynamique initiale [Pa.s]

θ	:	Coordonnée angulaire [degree]
ρ	:	Densité [kg/m ³]
ω :	:	Vitesse angulaire de l'arbre [rad/s]

Indice

i	Interne
ip	Indice du point d'intégration
s	Spécifique
t	Théorique
u	Utile

CHAPTER ONE

TRIBOLOGY

PR. NADIA BENDAOU

DR. MEHALA KADDA

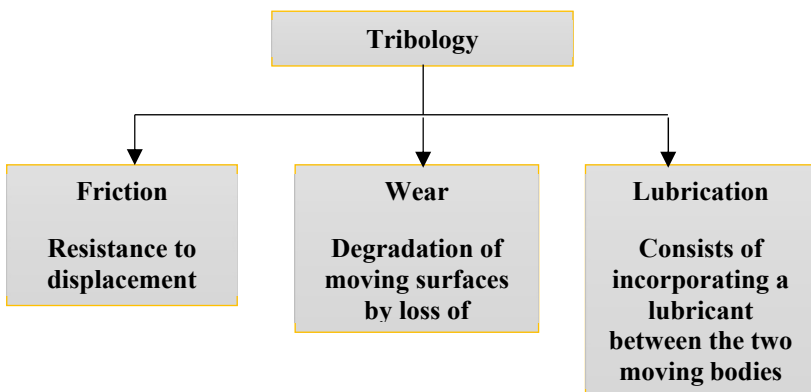
1. Tribology

1.1. General Comments on Tribology [1], [2]

1.1.1. Definition

Tribology comes from the Greek work *triben*, meaning the study of friction. Tribology is an integral part of the field of machine building.

Tribology is the science of the study of interactions of two surfaces in motion relative to each other. The field encompasses associated technology and all areas of friction and wear, including lubrication. It studies the interactions between contact surfaces, as well as those of the solids, liquids, and gases present between these surfaces. Its goal is the development of technologies aimed at optimizing friction.



1.1.2. Tribological systems

Applications of tribology are found wherever there is relative movement between two elements. For example, the cases of: plain bearings (between shaft and bearing); bearings (between the body of a bearing and balls, and between balls, tracks, and cages); and gears (between the profiles of pinions and toothed wheels). These pairs of elements are found in all machines, including engines, transmissions, compressors, and hydraulics. We again encounter tribology in metalworking (turning, milling, and drilling, etc.), but in this case, friction is found between the workpiece and the tool.

1.1.3. The purpose of tribology

Friction is one of the oldest known physical phenomena and certainly one of the most important in terms of its technological implications. This is the reason why the concept of tribology was introduced in 1966. The aim of tribology is to minimize material and energy losses due to wear and friction and successfully manufacture efficient mechanical and energy systems. The improvement it brings to operations involving moving surfaces (therefore, to the *tribological system*) is found in the greater efficiency and lifespan of machines.

The technical and economic aspects improved by tribology are:

- Performance and yield.
- Reliability and durability.
- Energy and component savings.
- Environmental impact

1.1.4. Optimization of the tribological system

Optimization involves three types of adaptations:

1. Decrease in contact: these are changes made with the aim to perform movements without any contact. For example, magnetic rails and slides.
2. Reduction of effort: these are measures that do not involve modifying the movements, reducing the effort necessary to carry the operation out. For example, the replacement of a plain bearing by a rolling bearing.
3. Structural changes to the system:

- a) Construction choices: during its design, the configuration and sizing of the system should be considered from a tribological angle.
- b) Choice of lubrication: the application and choice of the appropriate lubricant. In making this choice, there are two solutions: either the lubricant separates the moving surfaces, or it intervenes through a physical or chemical reaction.
- c) Choosing suitable materials: the selection of optimal pairs of materials or the use of optimized surface coatings.

1.1.5. Why is tribology so recent?

The relevant mechanisms have been around for a long time, but the study of tribology is relatively recent. This is due to a number of reasons:

❖Scientific reason:

Tribology has been of little interest to scientists because of the complexity of the material involved.

❖Technical reason:

- 1) The surface: the calculation of structures to improve strength has been accomplished for over a century.
- 2) Friction: friction is a primary source of wasted energy.
- 3) Wear: the durability of the parts involved is limited.
- 4) Unavailability: the cost of spare parts for a turbo alternator group is high.

1.2. Solid Contact Surface [5]

1.2.1. Definition

A solid contact surface is the interface between a solid and a second medium, which can be another solid, a gas (atmosphere) or a liquid.

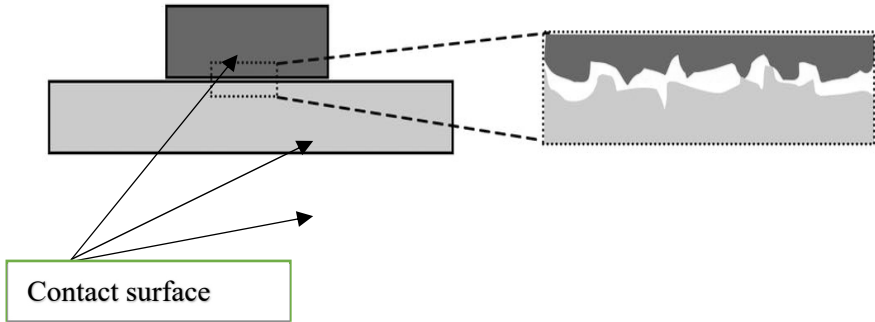
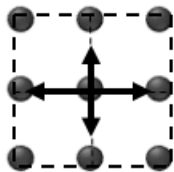


Figure 1.1. Contact surface.

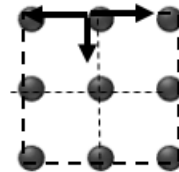
1.2.2. Surface structure

Solid materials have different physical properties at their surfaces and in their cores. An atom located inside the material is maintained in equilibrium by the forces exerted by neighbouring atoms. The crystallographic aspect of the core atom is presented in Figure 1.2b).

A surface atom is subjected to the forces of atoms located on the other side of the surface (see Figure 1.2a). This results in a reduction in interatomic distances, affecting the mechanical properties of the material's on the surface. In addition, the surface atoms are likely to interact with the atmosphere.



a) Core atom



b) Surface atom

Figure 1.2. Crystallographic aspect of surfaces.

1.2.3 Different layers of material

It is possible to identify the different layers (see Figure 1.3):

- **Layer of absorbed gas:**

This layer contains molecules of gas, water vapour, and oxygen absorbed by the solid. Its thickness ranges from 0.3 to 3 nm.

- **Oxide layer:**

Most materials react with oxygen to form an oxide layer with a thickness of 1 to 100 nm.

- **Beilby's layer:**

The crystal structure of this layer is different from that of the base material. It is 1 to 100 nm thick.

- **Deformed layer:**

This layer is subjected to residual stresses resulting from the manufacturing process. Its thickness varies from 1 to 100 μm .

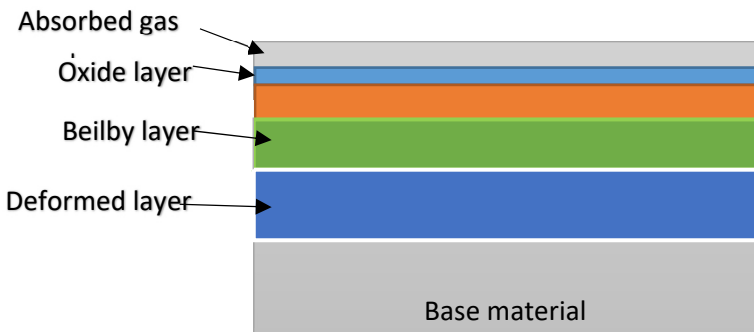


Figure 1.3. Different layers of the material

1.2.4. Surface topography

The surfaces present geometric deviations from their theoretical shape. Defects can be classified according to their wavelength “L”. Figure 1.4 illustrates the surface topography.

- If L is of the order of the size of the surface, it is a “form defect”.
- If $0.001 \text{ m} < L < 0.01 \text{ m}$, it is a “ripple fault”.

- If the length defect is smaller, it is “roughness”.

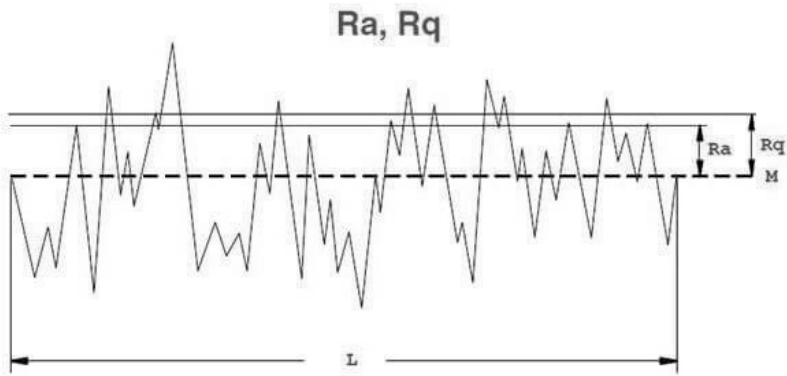


Figure 1.4. Surface topography.

1.2.5. Surface properties

a) Surface hardness

Hardness characterizes the ability of a material (surface) to resist deformation. It is generally measured by applying a force using a punch that is harder than the material tested to leave an imprint on the surface (see Figure 1.5).

Depending on the shape of the punch, there are different types of hardness:

- Brinell hardness with a ball (HB);
- Vickers hardness with a pyramid (HV);
- Rockwell hardness with a cone (HRC) or ball (HRB);
- Shore hardness (soft material).

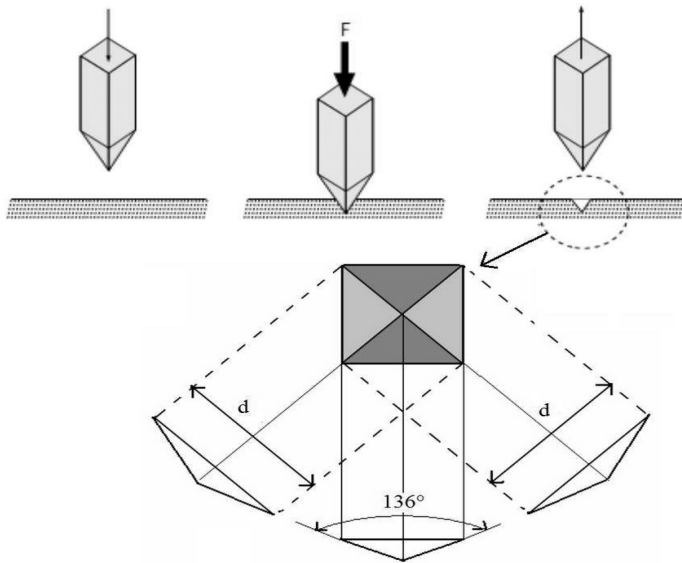


Figure 1.5. Measuring hardness.

Table 1.1. Material hardness

Material	HV (MPa)
Carbon steel	55-120
Stainless steel	140
Alumina (Al_2O_3)	1800
Diamond	10 000

b) Improvement of surface properties

To improve the performance and toughness of a surface, its hardness, energy, or resistance to corrosion is modified. The following techniques are used:

Surface treatment:

Consists of changing the chemical makeup or crystal structure of the material. For steel, there are several techniques to increase the surface hardness of the material:

- Nitriding: implantation of nitrogen atoms in steel.
- Carburization: implantation of carbon atoms in steel.
- Surface or total hardening: modification of the crystalline structure of steel.

Coating:

Consists of depositing a layer of a different material on the solid:

- **Hard deposit:** improving the hardness of the surface.
- **Soft deposit:** reducing the surface energy.

Surface energy (γ):

This energy comes from atoms on the surface which are free to interact with the environment. The higher this energy, the more easily the surface wears; the values of γ are low, but their impact is not negligible.

Table 1.2. Surface energy

Material	γ (mJ/m ²)
Metal	800-3000
Ceramic	50-120
Polymer	10-30

2. Friction [3]

2.1. Definition

Mechanical friction is a force that decreases the relative movement of two parts in contact (adhesion) and opposes movement, thus leading to a loss of mechanical energy. Two surfaces in contact are shown in Figure 1.6.

- When two bodies move in relation to each other in a viscous medium (air, water, oil), there is resistance to movement (kinetic friction force).

- When pushing an object resting on a horizontal plane, it is necessary to apply a certain force so that the object is put in motion. The sliding of this object on the plane causes a static friction force.

In both cases, the friction comes from the bond between the two surfaces or, more precisely, between the molecules in contact with the two surfaces.

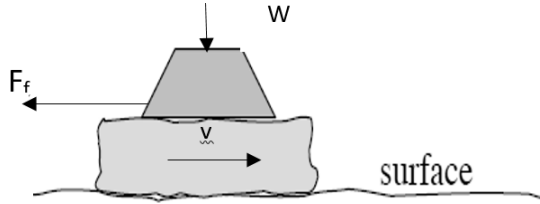


Figure 1.6. Surfaces in contact.

The friction factor is defined by the following relation:

$$f = \left| \frac{F_f}{W} \right| \quad (1.1)$$

- When $v \neq 0$, we speak of dynamic or kinetic friction

$$F_f = f \cdot |W| \quad (1.2)$$

- When $v = 0$, we speak of static friction

$$F_f < f \cdot |W| \quad (1.3)$$

During friction, power is dissipated:

$$P = F_f \cdot v = f \cdot W \cdot v \quad (1.4)$$

The dynamic friction force (F_f) can be the result of adhesive forces (F_a) and deformation forces (F_d), so that:

$$F_f = F_a + F_d = (f_a + f_d) \cdot W \quad (1.5)$$

where

f_a : coefficient of adhesive friction.

f_d : coefficient of friction of plastic deformation.

2.1.1. The contribution of adhesion to friction

Where solids are in contact, adhesive forces develop due to atomic interactions. To reinforce these connections, shear stress (τ_a) is necessary. The force that allows these bonds to be re-established is the force of adhesion, which is given by:

$$Fa = \tau_a \cdot A \quad (1.6)$$

where

A : total contact area (section).

The adhesive friction factor is given by the relation:

$$fa = \frac{Fa}{W} = \frac{\tau_a \cdot A}{W} \quad (1.7)$$

For a superficial elastic contact, the friction factor is given by:

$$fa = \tau_a \frac{2}{E} \sqrt{\frac{2\pi R}{\sigma}} \quad (1.8)$$

where

E : modulus of material elasticity.

σ : standard deviation between the two surfaces.

R : curvature radius.

If the asperity is deformed in a plastic way, the normal force ratio and the contact surface are equal to the duration (H) of a material and therefore the friction factor is expressed by:

$$fa = \frac{\tau_a}{H} \quad (1.9)$$

For an elastic contact, the adhesion friction factor is given by the relation:

$$fa = \frac{\tau_a \sqrt{2\pi}}{\psi H} \quad (1.10)$$

where

ψ : the flatness index, which is less than 1.

2.1.2. The contribution of plasticity to friction

In cases where a stiff material with a rough surface slides over a soft material with a smooth surface, the roughness of the hard surface will leave furrows. The presentation of the contact area between two solids is illustrated in Figure 1.7.

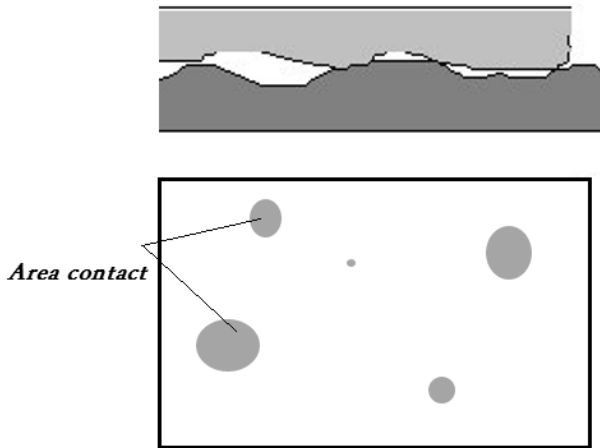


Figure 1.7. Contact area between two solids.

2.2. Friction for Different Types of Contact [4]

2.2.1. Contact between two revolving solids

Contact between both the revolving solids is shown in Figure 1.8.

* Contact area:

$$a = \left(\frac{3.P.R}{4.E^*} \right)^{1/3} m^2 \quad (1.11)$$

* Normal approximation of the two solids:

$$\sigma = \left(\frac{9.P^2}{16.R.E^{*2}} \right)^{1/3} m \quad (1.12)$$

* Maximum pressure under contact:

$$P_0 = \left(\frac{6 \cdot P \cdot E^{*2}}{\pi^3 \cdot R^2} \right)^{1/3} \quad Pa \quad (1.13)$$

where

R : curvature radius (m):

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} \quad (1.14)$$

$$\frac{1}{E^*} = \frac{1-\nu_1^2}{E_1} + \frac{1-\nu_2^2}{E_2} \quad (1.15)$$

where

ν_1 and ν_2 : the Poisson ratios of body 1 and 2.

E_1 and E_2 : the Young's moduli of body 1 and 2.

* Shear stress under contact:

$$\tau_s = 0.31 \cdot P_0 \quad Pa \quad (1.16)$$

*coefficient of friction

- elastic contact:

$$fa = \tau_s \cdot \frac{1}{E} \sqrt{\frac{2 \cdot \pi \cdot R}{\sigma}} \quad (1.17)$$

- plastic contact:

$$fa = \frac{\sigma_s}{H} \quad (1.18)$$

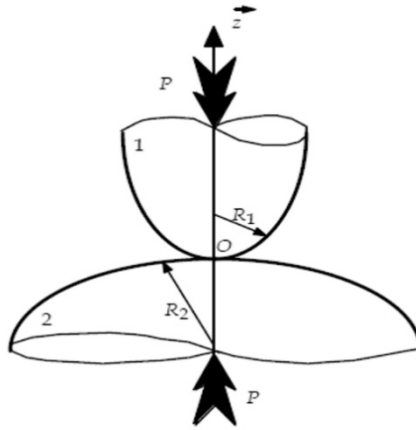


Figure 1.8. Contact between two revolving bodies.

2.2.2. Contact between two cylindrical solids along a single direction

Contact along a single direction between two cylindrical solids is shown in Figure 1.9.

* Contact area

$$a = \frac{4 \cdot R \cdot P}{\pi \cdot L \cdot E^*} \quad m^2 \quad (1.19)$$

L : Length of the cylindrical contact in (m)

* Maximum pressure under contact

$$P_0 = \left(\frac{P \cdot E^*}{\pi \cdot L \cdot R} \right)^{1/2} \quad Pa \quad (1.20)$$

* Maximum shear stress under contact $0.3 P_0$ to $0.78 a$

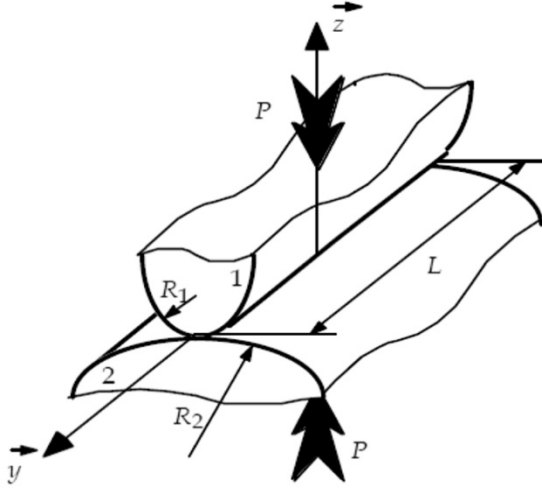


Figure 1.9. Contact between two cylindrical solids along a direction.

2.2.3. Point of contact between solids

The point of contact between solids is shown in Figure 1.10.

* Half-axes of the contact ellipse

$$a = e_a \left(\frac{11550 \cdot P}{E^* \cdot \sum \rho} \right)^{1/3} \quad (1.21)$$

$$b = e_b \left(\frac{11550 \cdot P}{E^* \cdot \sum \rho} \right)^{1/3} \quad (1.21)$$

where

e_a and e_b : the coefficients according to the principal curvatures of solids at points of contact

$$\sum \rho = \rho'_1 + \rho''_1 + \rho'_2 + \rho''_2 \quad (1.22)$$

where

ρ'_1, ρ''_1 et ρ'_2, ρ''_2 : the principal curvatures of solids at points of contact