Problems of Impact and Non-Stationary Interaction in Elastic-Plastic Formulations

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FOREWORD

This book is dedicated to my mother, Lyudmila Antonovna Popel, who was my first listener, and without her support I would not have been able to achieve and get everything I have. The book considers and compares three dynamic mathematical models: elastic, quasi-static and elastic-plastic. The problems of impact and non-stationary interaction of absolutely hard bodies and fine elastic shells are solved. The results are presented in the form of figures and tables. It is shown that the elastic model of impact is convenient to use to calibrate the numerical process in solving impact problems and non-stationary interaction in elastic-plastic formulation.

This was achieved using a new methodology of solving dynamic contact problems in elastic-plastic dynamic mathematical formulation. Specifically, for elastic-plastic dynamic mathematical modelling, a new methodology of destruction toughness $K_{Ic}(T)$ determination has been developed. The book includes the results of calculations of destruction toughness for irradiated RPV reactor steel. There is a comparison of the results of the impact of solid hard bodies and fine elastic shells and non-linear contact interaction developed using two approaches: 1) an elastic mathematical model and 2) an elastic-plastic mathematical model on the stage of elastic deformation.

This book considers and compares three dynamic mathematical models: elastic, quasi-static and elastic-plastic.

Solutions for contact problems are important for the determination of resource of the strength of the materials, crack resistance, plastic deformation and strain-stress states of construction such as aeroplanes, rockets, ships, trains, bearings, magistral gas- and oil-pipelines, large-scale metal constructions and constructions that have cylindric and spheric panels. New methodologies and approaches described in this book are useful for the precise solution of the problems of shock, thrust and impact and for the reliable simulation of dynamic contact processes. The newly developed methodology and approach of solving contact problems in dynamic elastic-plastic formulation offer the ability to design new composite reinforced and armed materials, such as the composite two-layer reinforced glass base material proposed in this book.

This book should prove of interest to scientists, students, post-graduate students and engineers.

V.R. Bogdanov

INTRODUCTION

It is necessary to determine the resource of the strength of the materials and crack resistance of constructions such as aeroplanes, rockets, ships, trains, bearings, magistral gas- and oil-pipelines, large-scale metal constructions and constructions that have cylindric and spheric panels.

A new methodology of solving dynamic contact problems, more precisely elastic-plastic dynamic mathematical formulation, is developed. On this basis it is possible to simulate more reliable and suitable non-linear, plastic processes in the material or, specifically, to calculate plastic deformation of the material. Constructions made of any material can have cracks. Constructions with cracks can survive for a hundred years; on the other hand, ones without cracks can collapse at short notice. The question is: does a material have brittle characteristics or not? The main parameter that describes the brittle characteristics of a material is destruction toughness or critical intensity factor $K_{\mathrm{I}c}(T)$. This book sets out a more precise elastic-plastic dynamic mathematical model in which a new methodology of destruction toughness $K_{\mathrm{I}c}(T)$ determination has been developed [21, 24, 28, 29, 31, 39, 54]. In the book are the results of calculations of destruction toughness for irradiated RPV reactor steel. There is a comparison of the results of the impact of solid hard bodies and fine elastic shells and non-linear contact interaction, developed using two approaches: 1) an elastic mathematical model and 2) an elastic-plastic mathematical model on the stage of elastic deformation.

Problems related to the non-stationary interaction of deformable and absolutely rigid bodies with the environment are of great practical and theoretical interest. The progress and development of modern technology leads to the need to study non-stationary and dynamic processes in various designs. Such processes are important in shipbuilding, aviation and rocketry. They occur, as a rule, in explosions, blows, shocks and impacts. The main elements of most designs are shells, plates and rods. Therefore, the study of the dynamic processes in such objects is of great interest.

Shock processes are encountered when solving a variety of problems. Their successful solution is associated with the harmonious interaction of various sciences: aero- and hydrodynamics, the theory of elasticity and plasticity, solids mechanics, destruction mechanics, the theory of shells and plates, applied and computational mathematics and others. The difficulty of solving problems of this kind is that it is necessary to jointly integrate systems of equations describing the motion of the body and the environment, when setting boundary conditions on unknown (moving) curved surfaces of the section. The position of these surfaces is determined in the process of solving. Therefore, precise solutions in this field of solids mechanics have mostly applied only to idealized, absolutely rigid objects.

Given the complexity of constructing analytical solutions to the problem of the penetration of deformable structures into the fluid (due to a significant change in the shape of contact and free surfaces, the emergence and development of cavitation zones in the liquid and elastic-plastic deformations in the structure material), both numerical-analytical and numerical methods are considered. The problem of the penetration of elastic shell structures into an elastic medium, when the boundary of the contact area lags behind the front of the waves that occur during impact, is also rarely studied.

The issues related to the non-stationary interaction of bodies and structures with a continuous medium are set out in a number of monographs: V.M. Alexandrov [1, 2], A.S. Volmir [64, 65], Sh.U. Galiev [72, 73], L.A. Galin [74, 172], W. Goldsmith [79, 80], V.T. Grinchenko [89], A.M. Guz' and V.T. Golovchan [90], A.M. Guz' and V.D. Kubenko [77, 91 – 93], R.M. Davis [94], K. Johnson [95], B.T. Diduh [96], J.A. Zukas, T. Nicholas and H.F. Swift [102], N.A. Kilchevsky [108], Y.V. Kolesnikov and E.M. Morozov [111], A.V. Kolodyazhny and V.I. Sevryukov [112], V.D. Kubenko [116, 120], E.N. Mnev and A.K. Pertsev [152], H.A. Metsaveer [149], G.I. Petrashenya [160], V.Z. Parton and P.I. Pearl [158], G.Ya. Popov [162], V.B. Poruchikov [170], S. Prasad [234], A.Ya. Sagomonyan [177, 178], L.I. Slepyan [182], L.I. Slepyan and Yu.S. Yakovlev [183], O.Y. Zhariy and A.F. Ulitko [100] and others, as well as in works by E.F. Afanasyev [8], A.E. Babayev [10, 11], A.G. Bagdoev [12], N.M. Belyaev [14], A.K. Efremov [13], F.M. Borodich [57–59], O.G. Goman [81], A.G. Gorshkov [60–62, 82–86], B.V. Kostrov [115], V.B. Poruchikov [169, 170] and D.V. Tarlakovsky [185, 186]. G. Kirchhoff was one of the first to consider problems relating to the unstable rectilinear motion of a rigid ball in an acoustic medium at a given speed in his 1876 monograph [211]. On the basis of the solution of the wave equation in a spherical coordinate system, A. Love [225] investigated the translational motion of a sphere in an acoustic medium in the presence of elastic force and a given velocity. The same solution is given in the monograph of G. Lamb [215]. Determining the loads at a given law of motion of a rigid body is the first step in solving the problem of the interaction of a moving obstacle within the environment. This problem, based on the Laplace transform over time, is considered in the monographs of B.V. Zamyshlyaev and Y.S. Yakovlev [101], E.I. Grigolyuk and A.G. Gorshkova [88], A.M. Guz', V.D. Kubenko and M.A. Cherevko [91], F. Muna [154], and W. Pao and S. Mou [233].

The problems of impact and penetration of absolutely rigid and elastic bodies into compressible and incompressible fluid are quite well studied. Solutions for the plane and axisymmetric problems of impact and penetration of stamps and elastic bodies into a compressible fluid were given in the works of V.D. Kubenko and his students [66–71, 84, 116–122, 124, 128–130].

The impact and penetration of stamps of different profiles into the elastic half-space were studied by V.D. Kubenko and S.N. Popov in [121, 133–135, 163–166].

The impact and penetration of elastic cylindrical and spherical shells into an elastic half-space were studied by V.D. Kubenko, S.N. Popov and V.R. Bogdanov in [15–17, 125–127, 134, 167, 168].

Analysis of the current state of research on the penetration of bodies into an elastic medium makes it possible to conclude that most of the work relates to the study of the penetration of non-deformable (absolutely solid) bodies. The penetration models used do not take into account a number of fundamental features of the dynamic interaction of the penetrating body and the elastic half-space. First of all, the possibility of deformation of the penetrating body is not taken into account, although taking it into account is extremely important when calculating penetration into the environment of thin-walled structures. The problems of impact and penetration of elastic shells into the elastic half-space have been little studied and the formulation, taking into account the influence of the rate of penetration of the shell and the corresponding rise of the medium due to it, has not been studied, despite these problems being of great scientific and practical interest. M.A. Lavrentyev, dealing with the impact of a projectile on armour, approached the elastic half-space as incompressible fluid.

D.V. Tarlakovsky studied the impact of elastic shells on an elastic half-space at the superseismic stage, when the rate of change of the boundary of the contact area is greater than the speed of longitudinal waves occurring in the half-space, and solved the unmixed boundary problem.

There are two major categories of numerical methods for solving partial differential equations: direct and indirect. Direct methods [217, p.3], which are known as strong-form methods, include well-known methods of finite differences [212], collocations with regularization [210, 217, 218], smoothing of hydrodynamic particles (smoothed particle hydrodynamics) [207, 219, 220, 226, 246], the gradient smoothing method [222, 223, 224, 244, 245], the discretisation method and the analytical method.

Indirect methods, known as weak-form methods, include finite element methods (MFE, mash dependent) [98, 208, 221, 222, 247]. The typical and most widely used weak form is the Galerkin weak form. Weak form equations are usually formulated in integral form, which implies the need to satisfy them only in the integral (average) sense, which is a weak requirement. The formulation of weak form methods is more general, and often these methods are more effective for less precise practice and engineering tasks.

There are two types of stability: spatial (existing in space) and temporal. The spatial stability of the method includes only the formulation of the method based on the finite spatial sampling model. When a method provides a stable solution for static problems, it is said that the method is spatially stable. The time-stable method provides a stable solution for dynamic problems and, therefore, includes precise formulation based on spatial and temporal discretization. However, when a stable temporal integrated circuit is used, a spatially stable method will not necessarily be stable when solving dynamic problems.

Catastrophic loss of accuracy occurs when the element mesh is significantly distorted. Therefore, the standard finite element method requires strict adhesion. In an isoperimetric element for which the so-called Jacobian matrix is defined, when the shape of the element is violated, the Jacobian matrix becomes poorly conditioned. The idea of smoothed finite element methods (S-FEM) [221, p.8] is to modify the combined deformation fields or to construct deformation fields using only displacement, hoping that Galerkin's model will give some good qualities. Such modification/construction of the deformation field can be carried out inside the element and more often outside the element, bringing information from neighbouring elements. Deformation fields satisfy certain conditions, and Galerkin's standard weak form must be modified accordingly to ensure stability and convergence. The question remains about the sufficiency of smoothing procedures to ensure a stable numerical solution.

According to the method of execution and formulation [180, p.9–10] of the basic equations of MFE or equations for individual finite elements, there are four main types of MFE: direct, variational, residual and energy balance.

The direct method is similar to the deformation method in the calculations of linear bearings. It is used to solve relatively simple problems and is convenient for clear geometric and mechanical values of individual approximation steps.

The variational method is based on the principle of stationarity of the functional. In the problem of solid mechanics, the functional is usually the potential energy of the system (Hellinger–Rayon, Hu-Vashizi). Unlike the direct method, which can only be applied to elements of a very simple type, the variational method is used with equal success on elements of both simple and complex types.

The residual method (weight residual method) is a generalized type of MFE approximation based on the differential equations of the problem under consideration. This method is used to solve problems for which it is difficult to formulate the functional or problems that do not have such a functional. The method of energy balance is based on the balance of different types of energy; it is used in thermostatic and thermodynamic analysis of a continuous medium.

In the mechanics of solid deformed bodies of the above types of MFE, a special place belongs to the methods of variation and residual, which in the area under discussion are two complementary methods of equal accuracy. The variational method is widely used, as expressions in the functional usually have a lower number of derivatives compared to the derivative in the corresponding differential equation of the problem, which allows one to choose interpolation functions from a wide palette of simple functions. The variational form of MFE is derived from the classical Ritza method and the residual method from the classical Bubnov–Galerkin method. In principle, from other variational methods, as well as from the residual method, it is also possible to derive the appropriate types of FEM. However, they are used much less often.

Unlike classical variational methods, in which the choice of interpolation functions depends on the configuration of the problem under consideration, in FEM this does not happen, as interpolation functions are defined exclusively within the framework of exclusive finite elements. Interpolation functions – a family of independent functions that describe an element, have zero values everywhere for all elements except the elements to which they relate. This is the main difference between FEM and the classical Rayleigh–Ritza and Bubnov–Galerkin methods, in which interpolation functions are determined for the whole domain.

The errors in FEM [180, p.119] by their nature can be twofold: sampling errors, which are the difference between the real geometry of the body and its approximation by a system of finite elements, and errors of interpolation functions, which appear due to the difference between the real field of unknown functions and their approximation by polynomials.

The type of finite element is determined by the degrees of freedom (and other parameters) attributed to its nodes [159, p.155]. By nodes we mean the geometric point of the area occupied by the finite element and in which is concentrated one or more degrees of freedom of this finite element. The corresponding class of finite elements could be called a class of finite elements with solid nodes (or finite elements of high precision) – an absolutely solid body of infinitesimal size. Finite elements with non-solid nodes are widely used. Elements of this type (for example, a triangular element with six degrees of freedom at the Cowper–Kosko–Lindberg–Olson node [203]) use as generalized degrees of freedom not only the values of the displacement function at nodal points and the values of its first derivatives in coordinates but also the values of the second derivatives of the same function. The motivation for introducing additional degrees of freedom is usually the desire to eliminate gaps in the displacement fields (eliminate incompatibilities) by increasing the rank of approximation of the quest functions, which is useful for improving asymptotic estimates of convergence. However, this can lead to complications and loss of accuracy in the linear transformation of the coordinates (the transition from the local coordinate system to the global system and vice versa).

The finite element methods, being mesh dependent, do not allow for the consideration of large deformations, unlike mesh-free methods of finite differences. It should be noted that the method of finite differences imposes the weakest requirements on the desired functions – they must be piecewise differentiable. The success and rapid development of finite element methods does not deny or diminish the importance of finite difference methods. The use of these methods in the problems of dynamics is strictly justified; the relations for estimating the error of the method are proved [103]. The impossibility of obtaining a guaranteed stable and convergent solution by the finite element method stimulates the further development and generalization of the finite difference method. A method [238] of generalized differences is being developed, which is a modification of the well-known and reliable method of finite differences with a variable partition step.

The study of impact in elastic staging continues to develop. Thus, in [148] the plane problem of the interaction of an absolutely solid drummer and an elastic isotropic homogeneous half-space at the supersonic stage within the framework of the theory of elasticity under conditions of rigid adhesion of contact surfaces is considered. The contact zone can be a multi-connected area. The problem of the dynamics of the interaction of the drummer with the half-space is reduced to the initial Cauchy problem for a system of quasi-linear differential equations.

In the dynamic problems of non-linear mechanics, the theory of plastic flow stands out among the existing models of the theory of plasticity. Among the most widely used fluidity conditions are two conditions: Treska–Sen–Venan and Maxwell–Huber–Mises–Genky, of which the second condition is more accurate, as shown by G. Genky at the First International Congress of Technical Mechanics. L.M. Kachanov made a great contribution to the development of the theory of plasticity and fracture [106, 107].

It is necessary to highlight problems with fast loading (such loading can be attributed to explosion). The contact process can be formalized by a rigid-plastic model. To date, studies on the dynamics of rigid-plastic structures cover a wide range of issues [156] and are detailed in a number of monographs [78, 97, 99, 173, 174] and reports [142, 175, 195, 209, 213, 216]. Since the impulse load is used in stamping products, the static and dynamic problems for rectangular, circular, annular plates and membranes with different shapes of the load impulse are well studied [156]. The condition of plasticity of Treska was mainly used. Regarding the bending of plane obstacles, most of the research concerns the problems of the axisymmetric deformation of circular and annular plates. A.A. Gvozdev [75] made modifications to generate an approximate solution for rectangular plates.

The work of V.N. Mazalov [141] is devoted to the study of the dynamic behaviour of annular plates with fixed contours loaded with a evenly distributed high-intensity explosive load. V.N. Mazalov and Yu.V. Nemirovsky [229] constructed a complete solution for the problem of dynamic bending of a rigid plastic annular plate with a free inner contour loaded with an evenly distributed explosive load. According to a single scheme, the problem is analysed for any method of fastening the outer circuit – from hinged resistance to clamping. In [231], D. Nipostin and A. Stanchuk constructed a complete solution for the problem of dynamic bending of an annular plate, similar to that studied by V.N. Mazalov [141], but for a load given by an arbitrary integrable time function and using the approximate Johansen plasticity condition. K.L. Komarov and Y.V. Nemirovsky [109, 110] considered the dynamic behaviour of rigid-plastic rectangular plates, taking into account their own weight and rectangular plates under the action of a rectangular moving impulse. To simplify the calculations, it was assumed that the linear hinges move uniformly within each interval of movement.

In the work of S. Bak and M. Muzhynsky [196], the dynamics of a hinged rectangular plate made of a rigid plastic material with the Huber–Mises–Genki plasticity condition were studied. The study of the dynamic behaviour of rigid plastic bodies in a resistant medium was carried out by G.S. Shapiro [190] for an infinite beam and by A.A. Amandosov [4], A.A. Amandosov and K.M. Stamgaziev [5], A.A. Amandosov and A.R. Uskombaev [6], M.M. Aliyev and G.S. Shapiro [3] and A. Kumar [214] for a circular hinged plate. In these works, the assumption of G.S. Shapiro was confirmed [190] that if the hinge lines are non-stationary, then the dynamics equations can be integrated in the case when the resistance of the medium depends on the speed of movement of the points. For plates, it is also concluded that the distribution of moments is independent of the resistance to the foundation.

In the monograph of V.A. Smirnov [181], an elastic-plastic solution to the distribution of plastic deformations both on the plane and on the thickness of square plates with mixed boundary conditions under a single applied, monotonically increasing static load is considered, and the load values corresponding to the limit state are obtained.

In monograph [156], a technique is developed based on the model of an ideal rigid plastic body that makes it possible to calculate the dynamic deformation of various shapes and fastenings of sheet metal structures from a single position. The results of the monograph are based on the authors' study of the dynamics of various single- and double-connected plates with arbitrary contour shape with different ways of fixing it. The influence of viscoelastic resistance to the foundation, as well as the variable thickness of plates and rigid inserts, is considered. Many important application problems have been solved. The results of the dynamic analysis are presented in a simple analytical form, convenient for further use. Throughout the work, comparisons are made of results calculated based on exact and approximate solutions.

Publications [235–237, 243] develop an approach to study the dynamic development of cracks in experimental samples based on the Rayleigh method, which approximates the dynamic processes in the beam using the so-called single degree of freedom (SDOF). This makes it possible to replace the dynamic model with a quasi-static one. In [235–237], the motion of the beam is described as a superposition of vibrational modes. To achieve greater accuracy of the model, the curvature of the drummer and supports is also considered. This method determines the dynamic stress intensity factor (DSIF) for the destruction process for 1.8 microseconds. In [243], an experimental-computational method for determining the dynamic stress intensity factor (DSIF) $K_{\mathrm{I}}(t)$ was proposed. The load and failure time of short compact specimens were determined experimentally. The single signal-response was calculated separately by the finite element method. DSIF was defined according to linear theory as a convolution of load and a single signal-response, while the critical value of DSIF corresponded to the moment of failure. The total period during which the destruction process was studied was 40 microseconds.

Attempts are being made to build a suitable model describing the motion of a fractured medium. The theory of asymmetric (moment stress and couple stress) elasticity was developed by E. Cosserat and F. Cosserat [202]. According to this theory, not only ordinary stresses but also moments must be determined in an infinitesimal neighbourhood. Another difference from the classical elasticity is that the deformation of the body is described by two vectors: the displacement vector $\mathbf{u}$ and the rotation vector ϕ. Cosserat's modern theory is developed for linearized two- and three-dimensional cases [191, 205, 230, 232, 239].

In bound moment theory, the vectors of rotation and displacement are not free but are related by a relation $\phi = (\text{curl}\,\mathbf{u})/2$. To distinguish between the cases of bound and unbound moment theories, Ehringen [205] proposed calling the unbound theory micropolar elasticity. In [192], an analysis of two crack models in an unbounded two-dimensional micropolar medium is carried out. The models relate to the problems of plane and antiplane deformation, respectively. In both cases, one of the three modes (forms) is not connected and the other two are connected. For a semi-infinite crack, the problem is reduced to a scalar and a second-rank Riemann–Hilbert vector with a Hermitian matrix of coefficients (couple stress is moment stress).

In [138, 147, 155], an approach was developed to use surface influence functions to solve non-stationary problems. In [155], non-stationary surface influence functions for an elastic-porous half-plane were determined and the problem of propagation of non-stationary axisymmetric perturbations from the circle surface of a Cosserat medium was investigated.

In the dissertation work [147], the peculiarities of solving the problem of diffraction of acoustic waves on unfixed elastic and deformable structures and non-stationary problems of diffraction of elastic and acoustic waves on inhomogeneous transversally isotropic inclusions of spherical shape, as well as a number of non-stationary contact problems based on smooth apparatus of surface influence functions, were researched.

In [18–52, 54, 201], the authors examine the approach of tracking the surface of plastic flow described by the Maxwell–Huber–Mises–Genki condition, based on the numerical solution of dynamic problems using the method of finite differences with variable pitch.

The problems of the impact of rigid bodies on deformable bodies and their collision remain relevant and are studied in various formulations. One of the most important areas of such research is to identify the features of the destruction of cut-notched beam specimens when they are destroyed on a three-point bend with a hammer. Relevant experiments make it possible to determine the much-needed fracture mechanics characteristic of the material – the fracture toughness or critical stress intensity factor associated with the stress intensity factor at the crack tip.

Because the process is dynamic and can be accompanied by significant plastic deformations, its study is a complex and multifaceted task that requires analysis of the impact on the test body, the dynamic interaction of the body with bearings and the process of destruction and its development. This topic is extremely important for the avionics, rocket and shipbuilding industries when examining the resource of the strength of the materials. [20, 23–52, 54] explore the problems of non-stationary shock, an impact interaction of an absolutely rigid plane drummer with a specimen notch-cut in the middle section in a dynamic elastic-plastic setting. In [37], the three-dimensional quasi-static problem corresponding to [23] in the elastic-plastic formulation was solved, and it was found that the stresses differ significantly from the stresses obtained from the solution of a similar problem in the dynamic elastic-plastic formulation. The publication [27] solves the problem of determining the stresses and the limit state in the plane strain state from the three-point bending of the beam specimen with a middle notch. In [25], a similar problem is explored of plane stress state under the criterion condition of the beginning of crack growth at the moment of moving the calculated maximum stress from the place of direct extension of the crack tip to a certain distance from it in order to ensure the existence of the maximum directly at the crack tip. In publications [26, 30], plane problems of stress and strain states with a crack are solved, the growth of which is controlled by a generalized local $\sigma_{\theta\theta}$ criterion of brittle fracture. In publications [28, 31], the fracture toughness of the material was determined based on the study of solutions of the plane strain state and spatial problems under the assumption that the crack is stationary, respectively. The proposed models made it possible in their development to significantly increase the level of adequacy of the obtained theoretical approaches. In [146], it was found that the quantitative characteristics of the necessary conditions for the formation of cold (brittle) cracks in the welding of low-alloy high-strength steels are quite clear when using a probabilistic model of brittle fracture with the Weibull distribution function, whose parameters generally depend on material microstructure and the percentage of diffuse hydrogen contained in the metal. There is no mention of the mathematical model and the problem from which the stresses used in the Weibull distribution relations are determined. This indicates that the probabilistic approach is quite universal and productive, but it is clear that if one uses a more accurate dynamic elastic-plastic formulation, the result will be more reliable. This is why in [43, 44] the three-dimensional process of crack growth with a straight front under the condition of shifting the maximum stresses from the crack tip and the local criterion of brittle fracture, respectively, were studied. In [131], the effect of non-stationary loading on the end surface of an elastic half-strip was investigated.

In [132], a supersonic impact is solved. The impact of a rigid cylinder is interesting as a limiting case of impact of elastic shells [11]. This book uses the approach in [17, 22, 120, 121, 123, 126, 132, 133, 136, 164, 165], which is based on a reducing of the initial equations of the dynamics of the shell-layer system to an infinite system of Volterra integral equations of the second kind. This makes it possible to effectively determine the numerical solution of the problems and reliably calculate the dynamic and kinematic quantitative characteristics that describe the collision process, depending on the magnitude of the initial impact velocity and the parameters of the shell and layer.

CHAPTER 1

FORMULATION OF ELASTIC AND ELASTIC-PLASTIC PROBLEMS

A thin elastic shell moving perpendicular to the surface of an elastic half-space $z \geq 0$ collides with an elastic half-space at a time $t = 0$. The shell begins to penetrate into the elastic medium at a rate $v_T(t)$, and the initial rate of penetration $V_0 = v_T(0)$. The shell's thickness h is much smaller than the radius R of the middle surface of the shell ($h / R \leq 0.05$).

Let us denote the undisturbed surface of the half-space by the sign Γ. Let us denote the part of the surface Γ that has not collided with the penetrating body $\Gamma_1(t)$; the rest of the surface Γ, through which penetration takes place $\Gamma_2(t)$, will be denoted and it will be called the contact area. Denote the surface bounding the penetrating body by $F(t)$. The area of the surface $F(t)$ that has not come into contact with the half-space is denoted $F_1(t)$, the rest is denoted by – $F_2(t)$ and we call it the contact surface. Equalities are valid for these surfaces: $\Gamma(t) = \Gamma_1(t) + \Gamma_2(t)$; $F(t) = F_1(t) + F_2(t)$.

The deformed free surface of the half-space is denoted by $\Gamma_1^*(t)$. We assume that there is no friction or just a slip condition between the elastic half-space and the penetrating body.

Boundary conditions will be as follows:

$$u_z\big|_{\Gamma_2(t)} = w_T(t) - f\left(\left\|\begin{matrix} x \\ r \end{matrix}\right\|\right) - w_0\big|_{\Gamma_2(t)} (\vec{n}, \vec{m}) -$$
$$- u_0\big|_{\Gamma_2(t)} \sqrt{1 - (\vec{n}, \vec{m})^2}, \qquad (1.1)$$

$$u_{\left\|\begin{matrix}x\\r\end{matrix}\right\|}\Bigg|_{\Gamma_2(t)} = w_0\big|_{\Gamma_2(t)}\sqrt{1-(\vec{n},\vec{m})^2} + u_0\big|_{\Gamma_2(t)}(\vec{n},\vec{m}), \quad (1.2)$$

$$\sigma_{zz}\big|_{\Gamma_1(t)} = 0, \quad (1.3)$$

$$\sigma_{\left\|\begin{matrix}zx\\zr\end{matrix}\right\|}\Bigg|_{\Gamma(t)} = 0, \text{ – slip condition} \quad (1.4)$$

$$f\left(\left\|\begin{matrix}x\\r\end{matrix}\right\|\right) = \begin{cases} f(x), & \text{if plane problem} \\ f(r), & \text{if axisymmetric problem} \end{cases}$$

where $\vec{m}$ and $\vec{n}$ are normal orts to the surfaces of the shell and half-space, respectively.

Since the law of motion of the shell is not known in advance, it is necessary to add the equation of motion of this shell to equations (1.1)–(1.4) and equations describing the dynamics of the elastic shell:

$$M\frac{d^2 w_T}{dt^2} = \iint\limits_{F_2(t)} \sigma_{zz}(t,s)(\vec{n},\vec{m})dF_2. \quad (1.5)$$

The refined model of fine shells of the S.P. Tymoshenko type makes it possible to consider the displacement and inertia of rotation of the cross-section of the shell. Perturbations spread in S.P. Tymoshenko-type shells at a finite speed. Therefore, the study of the dynamics of the propagation of wave processes in thin shells of the S.P. Tymoshenko type is an important aspect, just as it is important to study the wave processes of impact in the elastic medium into which the striker penetrates. The method of reducing the solution of dynamics problems to the solution of the second type of Volterra's infinite system of integral equations (ISIE) and the convergence of this solution are well studied. This approach has been successfully used for the study of problems of the impact of solids [133–135, 143, 164–166] and elastic thin shells of the Kirchhoff–Love type [15–17, 125–127, 136, 167, 168] on the elastic half-space and problems of the impact of solids [123, 132, 200] and elastic thin shells of Kirchhoff–Love type [22] on the elastic layer. An attempt was made [198, 199] to solve plane and axisymmetric problems of the impact of elastic thin cylindrical and spherical shells of the S.P. Tymoshenko type on the elastic half-space by the method of reducing the problems of dynamics to the solution of the second type of Volterra's ISIE. It is shown that this approach is unacceptable for the plane and axisymmetric problems of the impact of fine elastic shells on an elastic half-space studied in this book. Discretization using Gregory's methods for numerical integration and Adams for solving the Cauchy problem for the second type of Volterra's ISIE leads to the solution of a poorly defined system of linear algebraic equations: as the size of reduction increases, the determinant of such a system extends to infinity. This technique does not allow one to solve plane and axisymmetric dynamics problems for thin shells of the S.P. Tymoshenko type and elastic bodies [198, 199]. This shows the limitations of this approach [15–17, 22, 66–71, 116–136, 163–168, 200] and explains the need to develop other mathematical approaches and models [19–21, 23–54, 143–146, 227]. It should be noted that to calibrate the computational process in the dynamic elastic-plastic formulation at the elastic stage, it is convenient and appropriate to use the technique of summarizing the problems of dynamics to solve the second type of Volterra's ISIE [15–17, 22, 66–71, 116–136, 163–168, 200]. The solution of problems for elastic shells [63, 140, 150, 187], elastic half-space [9, 104, 151], elastic layer [137] and elastic rod [206, 241] was developed using the method of influence functions [87]. In [187], the process of non-stationary interaction of an elastic cylindrical shell with an elastic half-space at the so-called 'supersonic' stage (when the velocity of penetration is higher than the speed of the impact waves in the elastic half-space) of interaction is studied. This stage is characterized by exceeding the rate of expansion of the region of contact interaction by the rate of propagation of tensile-compressive waves in the elastic half-space. The solution was developed using influence functions corresponding to the concentrated force or kinematic effects for the elastic isotropic half-space, which were found and studied in [87].

§1.1. Impact of fine elastic cylindrical shells of Kirchhoff–Love and S.P. Tymoshenko types on an elastic half-space

A thin elastic cylindrical shell comes into contact with the elastic half-space $z \geq 0$ of its lateral surface along the generatrix of the cylinder at the time $t = 0$.

As shown in Figure 1, we associate with the shell a moving cylindrical coordinate system: $r\theta y'$: θ is the polar angle, which is postponed from the positive direction of the axis *Oz*. The axis $O'y'$ coincides with the axis of the cylinder.

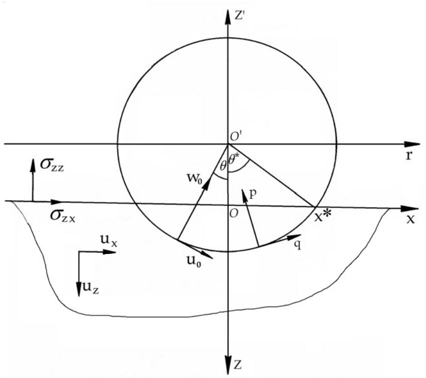

Fig. 1.1 Calculation scheme

Denote by $u_0(t,\theta)$, $w_0(t,\theta)$, $p(t,\theta)$ and $q(t,\theta)$ the tangential and normal displacements of the points of the middle surface of the shell and the radial and tangential components of the distributed external load acting on the shell, respectively. We connect a fixed Cartesian coordinate system *xyz* with a half-space, so that the *Oz* axis is directed inwards, the *Ox* axis is on the surface of the half-space and the *Oy* axis is parallel to the cylinder generator.

The physical properties of the half-space material are characterized by elastic constants: the modulus of volume expansion *K*, the shear modulus μ and the density ρ. The elastic medium with constants *K*, μ and ρ will correspond to the hypothetical acoustic medium with the same constants *K* and ρ ; thus, $\mu = 0$. By C_p, C_S and C_0 we mean the speed of longitudinal and transverse waves in an elastic half-space, as well as the speed of sound in a hypothetical acoustic medium corresponding to the considered half-space.

Let into the notation:

$$\beta^2 = \frac{C_S^2}{C_0^2} = \frac{\mu}{K}, \ \alpha^2 = \frac{C_p^2}{C_0^2} = \left(1 + \frac{4\mu}{3K}\right),$$

$$C_0^2 = \frac{K}{\rho}, \ b^2 = \frac{\beta^2}{\alpha^2} = \frac{3\mu}{3K + 4\mu}. \tag{1.1.1}$$

We introduce dimensionless variables:

$$t' = \frac{C_0 t}{R},\ x' = \frac{x}{R},\ z' = \frac{z}{R},\ u_i' = \frac{u_i}{R},\ u_0' = \frac{u_0}{R},$$

$$w_0' = \frac{w_0}{R},\ \sigma_{ij}' = \frac{\sigma_{ij}}{K},\ v_T' = \frac{v_T}{C_0},\ w_T' = \frac{w_T}{R}, \tag{1.1.2}$$

$$p' = \frac{p}{KR},\ q' = \frac{q}{KR},\ M' = \frac{M}{\rho R^2},\ (i, j = x, y, z),$$

where $\mathbf{u} = (u_x, u_y, u_z)$ is the displacement vector of points of the environment, σ_{zz}, σ_{xz} are the non-zero components of the stress tensor of the medium, M is the linear weight of the fine shell and $v_T(t)$, $w_T(t)$ are the velocity and displacement of the shell as a solid. In the future we will use only dimensionless quantities, so we omit the dash.

The elastic half-space and the shell are in a state of plane deformation. Given (1.1.2), from the system of basic equations of dynamics of arbitrary thin elastic shells based on Kirchhoff–Love's hypotheses, we obtain the equations of motion of a thin elastic cylindrical shell written with respect to dimensionless variables.

$$(1 + a_1)\frac{\partial^2 u_0}{\partial \theta^2} - \frac{\partial w_0}{\partial \theta} + a_1 \frac{\partial^3 w_0}{\partial \theta^3} = \beta_1 \frac{\partial^2 u_0}{\partial t^2} - \beta_2 q, \tag{1.1.3}$$

$$-\frac{\partial u_0}{\partial \theta} + a_1 \frac{\partial^3 u_0}{\partial \theta^3} + w_0 + a_1 \frac{\partial^4 w_0}{\partial \theta^4} = -\beta_1 \frac{\partial^2 w_0}{\partial t^2} + \beta_2 p,$$

$$a_1 = \frac{h^2}{12R^2},\ \beta_1 = \frac{C_0^2 (1 - v_0^2)\rho_0}{E_0},\ \beta_2 = \frac{R\rho(1 - v_0^2)K}{h\rho_0 E_0},$$

v_0, E_0, ρ_0 are Poisson's ratio, Young's modulus of elasticity and the density of the shell material, and p and q are the radial and tangential components of the distributed load acting on the shell, respectively.

Differential equations (of the S.P. Tymoshenko type), which describe the dynamics of cylindrical shells and consider the shear and inertia of rotation of the cross section, by virtue of (1.1.2) have the following form [178, p.87]:

$$\gamma_0^2 \frac{\partial^2 u_0}{\partial t^2} = \frac{\partial^2 u_0}{\partial \theta^2} + (1+a_4)\frac{\partial w_0}{\partial \theta} + a_4\Phi - a_4 u_0 + \beta_3 q,$$

$$\eta_0^2 \frac{\partial^2 w_0}{\partial t^2} = \frac{\partial^2 w_0}{\partial \theta^2} + \frac{\partial \Phi}{\partial \theta} - (1+a_3)\frac{\partial u_0}{\partial \theta} - a_3 w_0 + \beta_4 p, \quad (1.1.4)$$

$$\gamma_0^2 \frac{\partial^2 \Phi}{\partial t^2} = \frac{\partial^2 \Phi}{\partial \theta^2} - a_2 \frac{\partial w_0}{\partial \theta} - a_2\Phi + a_2 u_0,$$

where

$$\gamma_0^2 = \frac{C_0^2}{C_{02}^2},\ \eta_0^2 = \frac{C_0^2}{C_{01}^2},\ C_{01}^2 = \frac{E_0}{(1-\nu_0^2)\rho_0},\ C_{02}^2 = \frac{b_1^2 E_0}{2(1+\nu_0)},$$

$$a_2 = \frac{6(1-\nu_0)b_1^2 R^2}{h^2},\ b_1^2 = \frac{5}{6},\ a_3 = \frac{2}{(1-\nu_0)b_1^2},$$

$$a_4 = \frac{1}{a_3},\ \beta_3 = \frac{(1-\nu_0^2)K^2 R}{E_0^2 h},\ \beta_4 = \frac{2(1+\nu_0)K^2 R}{b_1^2 E_0^2 h},$$

where Φ is the angle of rotation of the normal section to the middle surface and b_1^2 is a factor that considers the distribution of tangential forces in the cross section of the shell.

The motion of an elastic medium is described by scalar potentials of equations φ and the non-zero component of vector potential ψ satisfying the wave equations:

$$\Delta\varphi = \frac{\partial^2 \varphi}{\alpha^2 \partial t^2},\ \Delta\psi = \frac{\partial^2 \psi}{\beta^2 \partial t^2},\ \Delta \equiv \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}. \quad (1.1.5)$$

Physical quantities are expressed in terms of wave potentials as follows:

$$u_x = \frac{\partial\varphi}{\partial x} + \frac{\partial\psi}{\partial z},\ u_z = \frac{\partial\varphi}{\partial z} - \frac{\partial\psi}{\partial x},\ u_y = 0,$$

$$\sigma_{zz} = (1-2b^2)\frac{\partial^2\varphi}{\partial t^2} + 2\beta^2\left(\frac{\partial^2\varphi}{\partial z^2} + \frac{\partial^2\psi}{\partial x\partial z}\right),$$

$$\sigma_{xz} = 2\beta^2\frac{\partial^2\varphi}{\partial x\partial z} + \frac{\partial^2\psi}{\partial t^2} - 2\beta^2\frac{\partial^2\psi}{\partial x^2},\ \sigma_{xy} = \sigma_{yz} = 0,$$

$$\Theta = \sigma_{zz} + \sigma_{xx} = 2(1-b^2)\frac{\partial^2\varphi}{\partial t^2},\ \sigma_{xx} = \Theta - \sigma_{zz}. \tag{1.1.6}$$

In a plane problem, it is natural to express normal displacements as $u_x = \frac{\partial\varphi}{\partial x} - \frac{\partial\psi}{\partial z},\ u_z = \frac{\partial\varphi}{\partial z} + \frac{\partial\psi}{\partial x}$. However, expressions (1.1.6) were used further in this book, because in this case the transition function $F_n(t)$ in (2.1.44) is the same as in the axisymmetric problem (§2.2) and, has a simpler form and is easier to calculate.

If the shear modulus $\mu = 0$, then the equations of motion of the elastic medium will be the equations of acoustics. Physical quantities will then be expressed in terms of displacement potential as follows:

$$V_x = \frac{\partial^2\varphi}{\partial x\partial t},\ V_z = \frac{\partial^2\varphi}{\partial z\partial t},\ V_y = 0,$$

$$\sigma_{ij} = \begin{cases} 0, & \text{if } i \neq j \\ -p = \dfrac{\partial^2\varphi}{\alpha^2\partial t^2} = \dfrac{\Theta}{2}, & \text{if } i = j,\ (i,j = x,y,z) \end{cases} \tag{1.1.7}$$

where V_x, V_y, V_z are components of the velocity vector of the points of the medium $\mathbf{V} = (V_x, V_y, V_z)$.

Consider the initial stage of the process of impact of elastic shells on the surface of the elastic half-space, when there is no plastic deformation and the amount of deepening of the shell into the environment is small. The problem of interaction of elastic shells with an elastic half-space is solved in a linear formulation; therefore, we linearize boundary conditions [120]: boundary conditions from the perturbed surface are demolished to undisturbed surfaces of deformable bodies.

As can be seen from Figure 1.1, the projection of the functions u_0, w_0, p and q on the axis $O'r$ and $O'z'$ will be equal:

$$\mathrm{pr}_{Z'} w_0(t,\theta) = w_0(t,\theta)\cos\theta,$$
$$\mathrm{pr}_{Z'} u_0(t,\theta) = u_0(t,\theta)\sin\theta,$$
$$\mathrm{pr}_{Z'} p(t,\theta) = p(t,\theta)\cos\theta,$$
$$\mathrm{pr}_{Z'} q(t,\theta) = q(t,\theta)\sin\theta,$$
$$\mathrm{pr}_{r'} w_0(t,\theta) = -w_0(t,\theta)\sin\theta,$$
$$\mathrm{pr}_{r'} u_0(t,\theta) = u_0(t,\theta)\cos\theta,$$
$$\mathrm{pr}_{r'} p(t,\theta) = -p(t,\theta)\sin\theta,$$
$$\mathrm{pr}_{r'} q(t,\theta) = q(t,\theta)\cos\theta.$$

Then, in the coordinate system *zOx*, movement u_z, u_x and stresses σ_{zz} and σ_{zx} at the surface points of the contact area will be written in the form:

$$u_z(t,x,0) = w_T(t) - f(x) - w_0(t,\theta)\cos\theta - u_0(t,\theta)\sin\theta, \quad (1.1.13)$$

$$u_x(t,x,0) = -w_0(t,\theta)\sin\theta + u_0(t,\theta)\cos\theta, \quad (1.1.14)$$

$$\sigma_{zz}(t,x,0) = -p(t,\theta)\cos\theta - q(t,\theta)\sin\theta, \quad (1.1.15)$$

$$\sigma_{xz}(t,x,0) = -p(t,\theta)\sin\theta + q(t,\theta)\cos\theta, \quad (1.1.16)$$

$w_T(t)$ is moving the shell as a solid; the function $f(x)$ describes the profile of the shell.

On the other hand, the radial and tangential components of the distributed load acting on the shell are expressed through normal and tangential stresses arising on the surface of the half-space in the contact zone.

$$p(t,\theta) = -\sigma_{zz}(t,x,0)\cos\theta - \sigma_{xz}(t,x,0)\sin\theta, \quad |\theta| < \theta^*, \quad (1.1.17)$$

$$q(t,\theta) = -\sigma_{zz}(t,x,0)\sin\theta + \sigma_{xz}(t,x,0)\cos\theta, \quad |\theta| < \theta^*, \quad (1.1.18)$$

where $2\theta^*$, as can be seen from Figure 1.1, is the value of the sector of the shell that has contact with the half-space.

The kinematic condition that determines the half-size of the contact area $x^*(t)$ will be written as follows:

$$w_T(t) - f(x) - u_z(t,x,0) - w_0(t,\theta)\cos\theta -$$

$$-u_0(t,\theta)\sin\theta = \begin{cases} 0, & \text{if} \quad |x| \le x^*(t) \\ \varepsilon < 0, & \text{if} \quad |x| > x^*(t) \end{cases} \qquad (1.1.19)$$

In this case, we assume that the contact region is simply connected, and this statement is equivalent to the fact that normal to the contact area stresses are compressive.

$$\sigma_{zz}\big|_{z=0} < 0, \quad |x| < x^*(t). \qquad (1.1.20)$$

Mathematically, we have a non-stationary mixed boundary problem of the theory of elasticity when displacements are given in the contact region and the rest of the half-space boundary is free of stresses. We will require compliance with the condition of complete slippage.

$$\sigma_{zx}\big|_{z=0} = 0, \quad |x| < \infty, \quad \sigma_{zx}\big|_{z=0} = 0, \quad |x| < \infty, \qquad (1.1.21)$$

Based on (1.1.8), the boundary conditions in the absence of friction in the contact zone can be formulated as follows:

$$\left.\frac{\partial u_z}{\partial t}\right|_{z=0} = v_T(t) - \frac{\partial w_0(t,\theta)}{\partial t}\cos\theta -$$

$$-\frac{\partial u_0(t,\theta)}{\partial t}\sin\theta, \quad |x| < x^*(t), \qquad (1.1.22)$$

$$\sigma_{zz}\big| = 0; \quad |x| > x^*(t), \quad \sigma_{zx}\big|_{z=0} = 0, \quad |x| < \infty.$$

The initial conditions for the potentials φ and ψ are zero:

$$\varphi\big|_{t=0} = \left.\frac{\partial\varphi}{\partial t}\right|_{t=0} = 0, \quad \psi\big|_{t=0} = \left.\frac{\partial\psi}{\partial t}\right|_{t=0} = 0. \qquad (1.1.23)$$

For the problem of the impact of the elastic shell on the elastic half-space, the velocity and displacement of the impact body is found from the equation of motion by its integration.

The equation of motion of the shell with mass *M* for the problem of impact with initial velocity V_0 has the form: