

The Mechanics of Smart Nanocomposite Sandwich Structures

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By

M.S.H. Al-Furjan, R. Kolahchi,
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FOREWORD

In this book, the mechanics of smart nanocomposite sandwich plates is presented. The main purpose of this book is to discuss the various theories of plate and of smart nanocomposite sandwich plates for mechanical analysis such as buckling, vibration and dynamic instability. This is a basic but essential book for MSc. and PhD. students for developing their project in the field of mathematical modeling and numerical analysis of structures. Here, various theories of plates such as classical, first order, third order and sinusoidal are presented. Then, different models are discussed for obtaining the effective material properties of nanocomposite structures such as the Mori-Tanaka, mixture, micro-electromechanical, and the Halpin-Tsai. In addition, the basic equations for smart materials (i.e., piezoelectric and magnetorheological fluid) are introduced. The governing equations for various examples are derived on the basis of the energy method and Hamilton's principle. The exact and numerical solutions are presented for the dynamic solution of the structures. The key findings of this book are the basic relations for smart, nanocomposite and sandwich plates as well as the mathematical modeling and mechanical analysis of them. This book can be developed further for beam and shell structures.

PREFACE

Nanocomposites are a new generation of multiphase or composite materials in which at least one of the constitutive materials dimensions is in nano scale. Obviously, nanocomposites have different and better properties with respect to composites due to the high surface-to-volume ratio of the reinforcing phase and high stiffness. For these reasons, nanocomposites have received much consideration among researchers due to the unique synergism between materials and the provision of new properties.

Recently, there has been intense attention given to piezoelectricity, motivated by developments made in smart piezoelectric structures. The coupling nature of smart materials has involved wide applications in electrical and electromechanical devices, such as sensors, actuators and transducers. In addition, magnetorheological fluids are sensitive liquids with respect to the magnetic field. In magnetorheological fluids under the magnetic field, the viscosity increases remarkably to the point of becoming a viscoelastic solid. The smart magnetorheological fluids have many practical applications in the design of wind turbines, absorbers and aerospace.

Motivated by these considerations, a comprehensive book for the mechanics of smart nanocomposite sandwich plates is a useful topic and of interest to researchers. This is a unique and engaging topic for all of the researchers, MSc. and PhD. students in the field of mechanical and civil engineering. This is because we have presented a complete book for the various practical subjects of smart, nanocomposite and sandwich plates, dynamic analysis, exact and numerical solutions, various theories of plates and nanocomposites, and mathematical modeling. For example, the civil engineering researcher can use this book for the mathematical modeling and numerical analysis of concrete foundations, slabs, frames and shear walls. In addition, the mechanical engineering researcher can use this book for the mechanical analysis and modeling of aircraft wings, marine structures, turbines, etc.

This book is divided into eight chapters. Chapter 1 describes nanocomposites as well as smart and sandwich structures and their properties. In Chapter 2, the basic relations of smart materials, and the various theories applied to

achieve nanocomposite properties and the modeling of plates, are discussed. Chapters 3-8 contain various practical examples for the mechanical analysis of smart nanocomposite sandwich plates.

BIOGRAPHIES

M.S.H. Al-Furjan is an associate professor of mechanical engineering, He earned his Ph.D. degree from Zhejiang University. He has rich research and teaching experience in different universities and institutes in China, Malaysia, and Iraq. His research involves structural mechanics, smart materials and structures, and numerical methods. He has been listed in the World Top 2% Scientists ranked by Stanford University.

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Our sincere thanks also go to our family members without their encouragement and support this book would not have been possible.

LIST OF ABBREVIATIONS

Abbreviations	Description
PMC	polymer matrix composites
PVDF	polyvinylidene fluoride
BNNTs	boron nitride nanotubes
CNTs	carbon nanotubes
CPT	classical plate theory
FSDT	first order shear deformation theory
S-SDT	second order shear deformation theory
TSDT	third order shear deformation theory
SSDT	sinusoidal shear deformation theory
HSDT	higher-order shear deformation theory
RVE	representative volume element
SLGS	single-layered graphene sheets
MWCNTs	multi-walled carbon nanotubes
NDNPS	nonlocal double-nanoplate-system
ESL	equivalent single-layer theory
MR	magnetorheological fluid
CNTRC	laminated carbon nanotube-reinforced composite
GPLs	graphene platelets
DCM	differential cubature method
PFRC	piezoelectric fiber-reinforced composite
FGM	functionally graded material
GDQM	generalized differential quadrature method

CHAPTER 1

SMART NANOCOMPOSITE MATERIALS

1-1 Introduction

Composites offer the advantageous characteristics of two or more combined materials with qualities that none of the other constituents possess. One component is named the reinforcing phase and the one in which it is surrounded is named the matrix. The matrix phase is mostly continuous. The reinforcing phase may be in the form of particles, fibers, or flakes. Examples of composite structures include polymer reinforced with fibers, concrete reinforced with steel bars, etc. [1].

The advantages of composite material with respect to other materials such as steel are its high strength and low weight. From the mechanical side, the advantage of composite depends on the application. For instance, the axial displacement of a rod subjected to an axial load is given by

$$u = \frac{PL}{AE}, \quad (1-1)$$

where u , P , L , A and E are axial deflection, axial load, length, cross section and Young's modulus of elasticity, respectively. Also, the mass of the rod (M) can be expressed as

$$M = \rho AL, \quad (1-2)$$

where ρ is the density of the material. Hence, we have

$$M = \frac{PL^2}{4} \cdot \frac{1}{E/\rho}, \quad (1-3)$$

in which E/ρ is called specific modulus and must be calculated for measuring the mechanical advantage of the composites. The other constant

is named the specific strength and is defined as the strength (σ_u) to density ratio. However, we have two parameters for determining the mechanical advantage of the composites which are

$$\text{specific modulus} = \frac{E}{\rho}, \quad (1-4)$$

$$\text{specific strength} = \frac{\sigma_u}{\rho}. \quad (1-5)$$

The two ratios are high in composite structures. For instance, the strength of epoxy reinforced with graphite would be the same as steel, but its specific strength is higher than that of steel (about three times). Also, the red cross section of epoxy reinforced with graphite could be the same as steel, but its mass is lower than that of steel rod (about one-third). This reduction in mass leads to reduced energy costs and material. Fig. 1-1 presents how composite structures and fibers rate with other traditional materials as a function of specific strength.

Note that in Fig. 1-1, the unit of specific strength is inches since the specific modulus and specific strength are defined as

$$\text{specific modulus} = \frac{E}{\rho g}, \quad (1-6)$$

$$\text{specific strength} = \frac{\sigma_u}{\rho g}, \quad (1-7)$$

where g is the gravity acceleration (9.81 m/s or 32.2 ft/s).

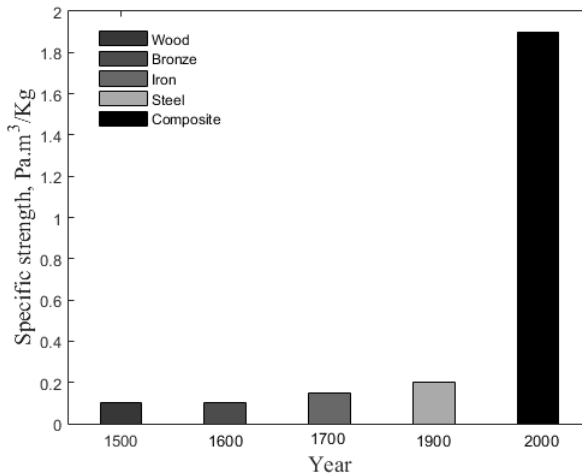


Fig. 1-1 Specific strength versus use time of materials [2]

The limitations and drawbacks in the use of composite structures include:

- ❖ The high cost for composites manufacture.
- ❖ The mechanical properties of a composite material are more complex than that of a metal material. In addition, composite structures are not isotropic and their characteristics are not the same in all directions. Hence, they need more material parameters.
- ❖ The repair of composite material is not a simple process with respect to that for metals. Sometimes critical cracks and flaws in composite materials may become invisible.
- ❖ Composite materials do not have a high combination of fracture toughness and strength with respect to metals. In Fig. 1-2, a plot is presented for fracture toughness as a function of yield strength for a 1in (25 mm) thick material. As can be seen, metals show an excellent combination of fracture toughness and strength with respect to composite materials.

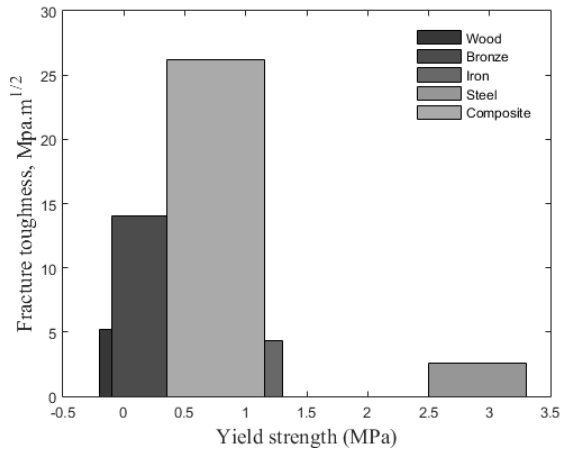


Fig. 1-2 Fracture toughness versus yield strength for ceramics, monolithic metals and metal-ceramic composite structures [3]

- ❖ Composite structures do not essentially give a higher performance in all the used material properties. In Fig. 1-3, six primary material selection factors: toughness, strength, joinability, formability, affordability and corrosion resistance are shown. If the values at the circumference are measured as the required normalized property level for a specific application, the shaded areas present the values provided by metals, ceramics, and metal-ceramic composite structures. Obviously, composite structures indicate better strength with respect to metals, but lower values for other used material constants.

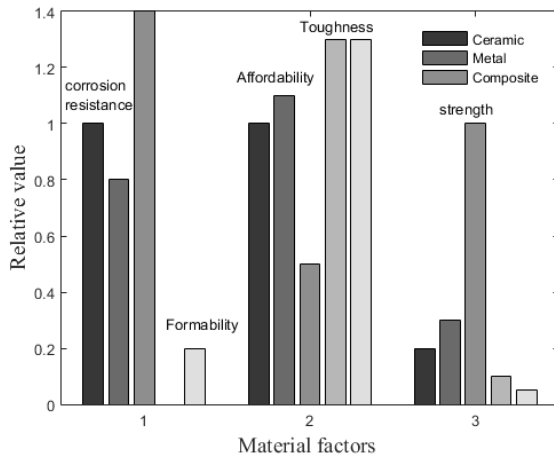


Fig. 1-3 Primary material selection factors for a hypothetical situation in ceramics, metals, and metal-ceramic composite structures [4]

1-2 Classification of Composites

Composite materials are classified by the type of matrix including metal, ceramic, polymer, and carbon or the geometry of the reinforcement counting flake, particulate, and fibers (Fig. 1-4).

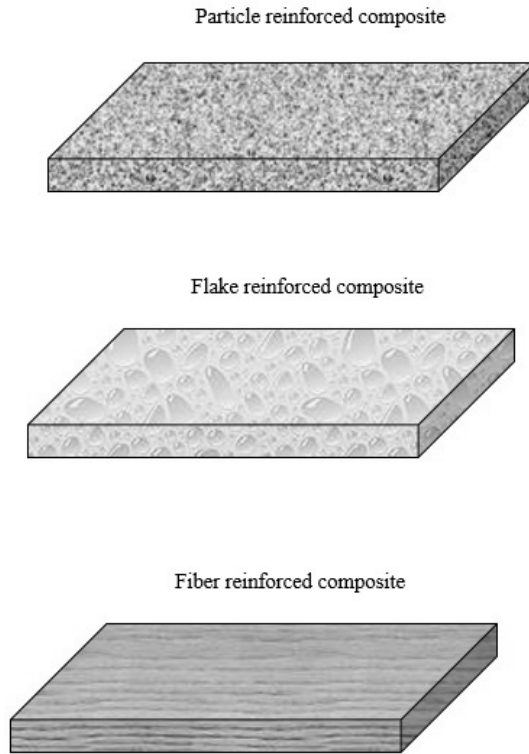


Fig. 1-4 Types of composite structures based on reinforcement phase

- ❖ Particulate composite structures involve particles embedded in matrixes such as ceramics and alloys. They are commonly isotropic since the particles are added randomly. These composites have advantages, such as high strength, oxidation resistance, operating temperature, etc. Typical examples are rubber reinforced with aluminum particles; aluminum reinforced with silicon carbide particles; and concrete reinforced with gravel, sand, and cement.
- ❖ Flake composite structures involve flat reinforcements of matrixes such as mica, glass, silver, and aluminum. These composites have good advantages including higher strength, high out-of-plane flexural modulus, and low cost. Hence, flakes cannot be oriented easily.
- ❖ Fiber composite structures involve matrixes reinforced by long (continuous) or short (discontinuous) fibers. Fibers are mostly

anisotropic, such as aramids and carbon. Examples of matrixes are metals such as aluminum, resins such as epoxy, and ceramics such as calcium-alumino silicate. Continuous fiber composites are very important with practical application and can be found by the various types of matrixes such as metal, polymer, carbon, and ceramic. The fundamental units of composites with continuous fibers are woven fiber or unidirectional laminas. Laminas are arranged on top of each other at different angles to form a multidirectional composite laminate.

1-3 Polymer Matrix Composites

The most advanced common composites are polymer matrix composites (PMCs) containing a polymer (e.g., polyester, epoxy, and urethane) reinforced with thin diameter fibers (e.g., aramids, graphite, and boron). It is because these composites have high strength, low cost, and simple manufacturing principles. For instance, epoxy reinforced with graphite composites are approximately five times stronger than steel. The main disadvantages of PMCs are high coefficients of moisture and thermal expansion, low operating temperatures, and low elastic constants in certain directions.

1-4 Nanocomposites

Nanocomposites contain materials in nano scale (10^{-9} m). The accepted range in nano scale for the nanocomposite structures is less than 100 nm for the matrix or reinforcement phases. At nano scale, the material properties are different from those of the macro material. Usually, advanced composite structures have constituents in microscale (10^{-6} m). By having nanometer scale materials, most of the resulting composite properties are better than the ones at the micro or macro scale. Not all characteristics of nanocomposites are better; in some cases, impact strength and toughness can reduce. The practical applications of nanocomposites are aerospace, packaging applications for the military and oil pipelines since the nanocomposite films present an improvement in properties such as heat distortion, elastic modulus, and transmission rates for water vapor and oxygen.

1-5 Smart Nanocomposites

Smart nanocomposites are a new class of composites of which one of the reinforcement or matrix phases, or both of them, is smart. Smart nanocomposites used in recent years, have received an intense interest among researchers due to the smart and unique properties between materials. Polyvinylidene fluoride (PVDF) is a perfect piezoelectric matrix due to its properties such as excellent dimensional stability, high strength, abrasion and corrosion resistance, thermoplastic flexibility, and good mechanical properties at elevated temperatures. It has, however, found numerous applications in nanocomposites in a wide range of industries such as the petrochemical, oil and gas, electronics, automotive, wire and cable, and construction industries. Boron nitride nanotubes (BNNTs) utilized as the matrix reinforcers, apart from having good electrical, mechanical, and chemical properties, show more resistance to oxidation and high temperature resistance than other conventional nano-reinforcers, such as silica nanoparticles, carbon nanotubes (CNTs), etc.

1-6 Sandwich Structures

Sandwich structures are a kind of composite material with two or more separate components with different properties which, when combined, lead to a high performance structure. Compared to monolithic composites—which contain an intimate mixture of fibers (Kevlar, glass, metal, carbon, etc.) embedded in a continuous matrix (e.g., thermoset resin or thermoplastic)—sandwich structures have a discrete structure with a core integrated with skin material. The core usually has high shear strength and compression while skin material generally has a high stiffness. When the core and skins are bonded together, this combination provides the sandwich structure with a high flexural modulus.

References

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CHAPTER 2

BASIC THEORIES

2-1 Introduction

Plates are two types of structural elements which can be used in many mechanical, civil and aerospace applications. The thicknesses of these structures are very small compared with their other dimensions. However, one can say that a shell can be resultant from a plate by means of forming the middle surface as a singly or doubly curved surface. In this chapter, most of the plate theories applied in many mechanical problems are presented.

2-2 Strain-Displacement Equations

When a continuous body is subjected to a physical action, it is assumed that it changes continuously. It should be mentioned that changes are continuous which means no fracture is considered. Consider an elastic body at time $t = 0$, in which an arbitrary point of the deformable body, such as A, occupies a position \mathbf{X} in the reference configuration. After the body has deformed, point A changes its position and moves to point B; it occupies a new position \mathbf{x} . In continuum mechanics, two regular descriptions of motion and deformation (Lagrangian and Eulerian) can be applied. In the referential or Lagrangian description, the motion of the body refers to the original undeformed configuration; that is, the one which the body occupies at time $t = 0$. The Lagrangian description is also called the material description, in which the current coordinates x_i are written in terms of the reference coordinates X_i and time t as follows:

$$\mathbf{x} = \mathbf{x}(\mathbf{X}, t), \mathbf{x}(\mathbf{X}, 0) = \mathbf{X}. \quad (2-1)$$

In the Eulerian or spatial description, the motion of the body is referred to as the current or deformed configuration. Therefore, a typical variable χ is depicted in terms of the current position as:

$$\chi = \chi(\mathbf{x}, t), \mathbf{X} = \mathbf{X}(\mathbf{x}, t). \quad (2-2)$$

When the configuration of a body changes due to external forces, the displacement vector \mathbf{u} is written as:

$$\mathbf{u} = \mathbf{x} - \mathbf{X}. \quad (2-3)$$

In the present book, the Lagrangian description is applied, due to its simplicity and convenience. Hence, in order to obtain the Lagrangian (or Green's) strain tensor the components of the displacement vector u_i are functions of x_i .

Consider two neighboring points $A(X_1, X_2, X_3)$ and $A'(X_1 + dX_1, X_2 + dX_2, X_3 + dX_3)$ which are connected together with an infinitesimal line. The square of the line length can be calculated in the undeformed configuration as follows:

$$\overline{AA'}^2 = dS^2 = dX_1^2 + dX_2^2 + dX_3^2. \quad (2-4)$$

After deformation, points A and A' change their position and become $B(x_1, x_2, x_3)$ and $B'(x_1 + dx_1, x_2 + dx_2, x_3 + dx_3)$ in the deformed configuration. The square of the line length connecting B to B' is written as:

$$\overline{BB'}^2 = ds^2 = dx_1^2 + dx_2^2 + dx_3^2. \quad (2-5)$$

The differentials dx_i can be written with respect to the original coordinate system X_i as:

$$dx_i = \frac{\partial x_i}{\partial X_1} dX_1 + \frac{\partial x_i}{\partial X_2} dX_2 + \frac{\partial x_i}{\partial X_3} dX_3. \quad (2-6)$$

Using Eq. (2-6), Eq. (2-5) can be rewritten as follows:

$$ds^2 = \sum_{k=1}^3 \sum_{i=1}^3 \sum_{j=1}^3 \frac{\partial x_k}{\partial X_i} \frac{\partial x_k}{\partial X_j} dX_i dX_j. \quad (2-7)$$

Using Eqs. (2-4) and (2-7), the difference between the squares of the lengths of BB' and AA' can be written as follows:

$$ds^2 - dS^2 = \sum_{k=1}^3 \sum_{i=1}^3 \sum_{j=1}^3 \left(\frac{\partial x_k}{\partial X_i} \frac{\partial x_k}{\partial X_j} - \delta_{ij} \right) dX_i dX_j, \quad (2-8)$$

in which δ_{ij} is the Kronecker delta. Considering Eq. (2-8), the Green's strain tensor \mathcal{E}_{ij} is obtained as:

$$ds^2 - dS^2 = 2 \sum_{i=1}^3 \sum_{j=1}^3 E_{ij} dx_i dx_j. \quad (2-9)$$

The following equation may be written using Eq. (2-7) as follows:

$$E_{ij} = \frac{1}{2} \left(\sum_{k=1}^3 \frac{\partial x_k}{\partial X_i} \frac{\partial x_k}{\partial X_j} - \delta_{ij} \right). \quad (2-10)$$

Eq. (2-1) can also be rewritten as follows:

$$u_k = x_k - X_k \Rightarrow \frac{\partial x_k}{\partial X_i} = \frac{\partial u_k}{\partial X_i} + \delta_{ki}. \quad (2-11)$$

Therefore, the Green (Green-Lagrange) strain tensor can be obtained by substituting Eq. (2-11) with Eq. (2-10) as: